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THE UNIVERSITY OF ALBERTA
STATISTICAL ANALYSIS OF PLAN AND DEPTH
FORMS OF THREE RIVERS

by



sperare:beth

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
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IN

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EDMONTON, ALBERTA

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THE UNIVERSITY OF ALBERTA
FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Statistical Analysis of Plan and Depth Forms of Three Rivers", submitted by ~~aperare:beth~~ in partial fulfillment of the requirements for the degree of Master of Science.

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ABSTRACT

Data for planform (meandering) and depth profiles for three Canadian rivers (Beaver, Alberta; Kootenay, B.C.; South Nahanni, N.W.T.) have been collected in an attempt to better understand river behavior. Earlier workers attempted to explain such forms by secondary flows, transverse oscillations, and the earth's rotation, but did not predict river behavior well. More recently, stochastic models produced variance-minimizing sine-generated curves which approximated meandering. Spectral analysis has been applied by Speight (1965) and others to analyze meanders, and by Nordin (1966) and other to analyze bedforms. Stochastic processes and variance spectrum analysis are reviewed.

Collection techniques for the data acquired are outlined, as are the relevant physical geographic characteristics of the areas and rivers studied. Methods of digitizing, correcting, storing and analyzing the meander and depth series are presented.

All series are at least weakly self-stationary. Digitizing and analytic techniques used are efficient and inexpensive. Simple statistics can reveal general trends in such data, but spectral analysis shows: these pronounced (down-path) wavelengths are between 6-20 river widths; most wavelengths occur as multiples of (π x width); and most spectra resemble those characteristics of a second-order autoregressive process.

Recommendations for future research include: improvement in types of data collected; improvement in resolution of data actually used in analysis; better absolute location when collecting data; better data

analyses; and expansion of the research to include 2-D cross-spectral analysis of bedforms.

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A presentation of the abstract, as immediately precedes this section, introduces the question of a discussion of the concrete. The more tangible forces entirely essential to the production of this thesis have combined as a fine-grained matrix of knowledge and skill, lending unity and polish, substance and completion to the larger, more easily identified units of this work. It is truly a pleasure to acknowledge the concrete assistance of those who have so paved the way towards full-circle of this phase of my life, my work.

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Research Council of Alberta provided other essentials for the field work conducted in the course of this study. Boats, sounding equipment, and the skilled manpower necessary for the collection of depth data on the Beaver and Kootenay Rivers were made available for this work through the Research Council of Alberta, via the support of its research officers, Mr. C. Neill and Dr. R. Gerard. The navigation skills of Mr. Herman Schultz, also of the Research Council, provided an extremely accurate thalweg depth examination of the Beaver, and provided safety of equipment and persons involved in the more demanding Kootenay River survey.

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LIST AND EXPLANATION OF SYMBOLS IN THIS THESIS

λ	wave length
m	lag specification
n	total number of points in a series
μ	series mean
$R_{xy}(m)$	cross-covariance function
$\rho_{xy}(m)$	cross-correlation function
$I_{xy}(\omega)$	cross-spectrum
$Q_{xy}(\omega)$	quadrature spectrum (imaginary component of the cross-spectrum)
$P_{xy}(\omega)$	cospectrum (real component of the cross-spectrum)
$\frac{P_{xy}}{Q_{xy}}$	measure of the phase relations
$\phi(f)$	phase angle
Coh	coherency spectrum
Δx	sampling interval
ϕ	angular deviation along the flow path
$+\phi$	"relative curvature" - changes in flow direction assigned either positive or negative values to specify right vs. left turns in flow
$ \phi $	"absolute curvature" - absolute value of angular deviation - flow change considered only with respect to magnitude, ignoring specific direction
R	radius of curvature
w	river width, defined further in context

d,D river depth, defined further in context

series A data for South Nahanni River, upstream survey

series B data for South Nahanni River, downstream survey

series C data for Kootenay River, upstream survey

series D data for Beaver River, upstream survey

INTRODUCTION

A. The Problem

Effective control of the behavior and form of natural systems can only be achieved through sufficient understanding of the various processes interacting to maintain the systems concerned. A system must be described, defined, and understood before it can be safely tamed or made more productive. Much remains to be learned about the river, and the potential contained in such knowledge is great.

The tendency of all rivers to establish and maintain curved sections of channel in plan continues to fascinate fluviologists. The process of meandering is neither fully described nor fully explained. Even less data is available on the "meandering" pattern assumed in the vertical dimension by the river's bed. Laboratory work in fluvial geomorphology allows a variety of hypotheses to be tested against control situations. Nature does not offer the same advantages, and relatively little field data of great detail exists for natural rivers. Correspondence of lab and field studies must continually be tested toward the building of theory in fluvial geomorphology.

Additional expansion of the potential utility of detailed field data, through the sophistication of methods in data collection and treatment, must be maintained as a major aim of work in fluvial geomorphology as theory is established. New techniques may be introduced from other fields or developed within the discipline, but the level of analysis of data must constantly be advanced.

Briefly, then, both the quality and quantity of data gathered towards solution of the problem of fluvial meandering must be improved. Analytic techniques must also be upgraded so that the level of understanding is increased. Comparative utility of data and analyses must be maximized within the field.

B. Purpose of Research

Although it is possible to obtain continuous depth profiles of navigable rivers, previous studies in fluvial geomorphology have seldom exploited the potential of such data. Consequently, comparative studies using continuous depth profiles for rivers, covering a variety of channel types, bank and bed materials, geologic histories, river sizes, sediment loads, slopes, and discharges are undertaken for the first time at this scale in this thesis. Among the initial purposes established for this research were thus:

1. to gather continuous depth profiles for rivers of different channel types, bank and bed materials, geologic history, size, sediment load, slope, and discharge, and
2. to digitize, store, and so make available the data collected, for use by other researchers, government agencies, or any concerned individuals or groups.

Comparative potential of this research was also considered important. Dr. M. Church of the University of British Columbia, is carrying out complimentary research, and he is primarily interested in scales of turbulence operating within fluvial systems. He has collected data on very small rivers (small mountain streams) and very large rivers

(the Mackenzie River). Data on rivers of intermediate size will thus complete the initial examinations of river plan and depth inter-relations with respect to scales of turbulence:

3. to accumulate data on rivers of intermediate size for use in future research on plan and depth inter-relations for rivers of all sizes; conducted in conjunction with Dr. M. Church of U.B.C.

Unless analysis of data can handle the detail gathered, its value remains locked in potential utility. Superiority of analytic techniques with respect to quantity of data input and quality of information output is sought. More objective analysis must also be attained. Techniques employed in previous studies, where "typical" meanders were described, must be improved upon. Variance spectrum analysis provides sophistication not yet fully explored in conjunction with fluvial geomorphologic studies. Many problems are yet to be both discovered and subsequently solved in order for the technique to become established as a form of communication and adequate basis of comparative study in the field. Thus, other purposes underlying this research include:

4. to attempt minimization of subjectivity, inherent in previous analyses of meander planform, and provide descriptions of the rivers studied on this basis.
5. to explore the utility of spectral and cross-spectral analytic techniques for data collected along the path of flow in rivers.
6. to derive information from the data collected on both the planform geometry and bedform configuration of rivers, through the use of

spectral and cross-spectral as well as other "less-sophisticated" techniques, and to explore the inter-relations of planform and depth, in order to provide descriptions of the rivers studied.

C. Scope of the Thesis

Availability of equipment, finances, and time constituted the major constraints encountered during research for this thesis. The use of depth recording machinery and boating equipment had to be arranged in co-operation with external organizations, and in this regard, research schedules became fixed rather than spontaneous. This limited examination of processes to those operating only at specified times, chosen rather arbitrarily, within the river systems studied. The study of processes operating at specific discharge stages (eg. bankfull) was thus beyond the scope of this study.

Adequacy of first-hand knowledge of study areas also limited the scope of the study. Thalweg soundings were not recorded during the South Nahanni survey, as it was believed that this would create problems with respect to safety of crew and equipment. It seems, in retrospect, that thalweg depths could actually have been obtained, but this knowledge was not available prior to the study.

The scope of the field of research conducted for this study thus allowed only single samplings of the depth profiles of three accessible rivers. Lessons derived from the experience gained in the course of the research completed are listed in Chapter 5, under "Recommendations for Future Research".

CHAPTER I

THEORETICAL CONTEXT OF THIS STUDY

Early studies of the morphogenesis of fluvial landscapes immediately indicated the fluvialist's fascination with and underlying curiosity concerning the meandering form so consistently assumed by rivers. The physical behavior of the dynamic systems that mold the sinuosities so characteristic of fluvial sceneries is still not fully understood. Satisfactory statistical, geometrical, and descriptive definitions of the meandering form have not been attained in any wide sense within the discipline, but various approaches have each gained the acceptance and support of a number of fluvial geomorphologists during the development of the discipline.

Classical explanations of the initiation, persistence, and form of fluvial meandering have varied widely in their approach to the problem. Examinations of form have been attempted in order that process might be understood, and process has been examined to explain form. As well, many physical properties have been examined simultaneously. Though many relationships have been established as hydraulic laws, and though many fluvial processes are now understood as complex components of river behavior, no single set of theories and no single model sufficiently explain the intricacies of meandering.

A full appreciation of the theoretical problems and challenges involved in the attempt to explain the meandering tendency of rivers is best gained through an examination of the theoretical development to date. Many problems have been recognized in theory, and many have

been solved. The rigor of the definition of generating processes has been greatly increased and models have been substantially improved. Much of the subjectivity of the early analyses has been eliminated through sophistication of methodology and research technology. As theory has developed in other disciplines, fluvial geomorphology has grown, incorporating concepts of turbulence, and incorporating many ideas initiated in hydraulic engineering. Thus, although the meandering of rivers is not yet fully explained, considerable theoretical advances have been made.

The principal theoreticians who have contributed insight into the problem of meandering are discussed below. In order to provide as objective an introduction as possible, contributions are presented in the terminology of the authors. Some criticism is then offered as the development is traced.

A. Early Models

Boussinesq (1872) offered an analogy between the flow in a bent pipe and that in a curved section of open channel. Thompson (1876) developed the physics of this concept and presented his own impression of flow, theorizing that, stretched out, a stream was really screw-shaped so that flow was characterized by "transverse circulation". Though Thompson early discovered the phenomenon of transverse circulation, (later to become known as secondary flow), his theories did not achieve widespread support for some time. Nearly all early work in the fields of fluvial geomorphology and river control, extending into the early twentieth century, was based on the assumption that streamflow proceeded as an infinite number of flow lines parallel both to each other and to their confining banks.

Lelyavskii's work on the Dnieper basin (1893-1904) led, from observations of the velocity and direction of flow at different points in the river, to the conclusion that two distinct types of flow co-existed within channels. "Converging flow" caused scour while "diverging flow" left channel deposits. Lelyavskii later proposed that a natural channel has its adjacent flow lines parallel neither to each other nor to the banks, except in the flow zone near the banks. (Kondrat'ev, 1962)

Fargue (1908) conducted fluvial studies based on the Garonne River. His basic observation was that the maxima of channel depth and channel curvature are not coincident, but rather, that the point of greatest depth in a channel is displaced downstream from the cross-section of greatest curvature. This phenomenon has since been confirmed (Chang, 1969; Leopold and Wolman, 1960). Fargue also observed analogous lags between points of inflection of channel curvature and shallows of minimum depth. He concluded that variations in channel curvature determine the stability and roughness of the channel such that curvature changes would directly induce changes in the channel profile. Greater curvatures were expected to produce deeper scour farther downstream. (Kondrat'ev, 1962)

At the turn of the century, then, fluvial theory was already exploring the existence of connections between channel curvature and depth, and the notion of transverse circulation was available as a possible explanation of the initiation of meandering.

B. Process Analysis

Another school of fluvialists was contemporaneously concerned with the association of river meandering and the rotation of the earth.

Eakin (1910) wrote of "unbalanced lateral erosion" due to the earth's rotation, and even as late as 1943, Quraishy (1943) championed this theory. The theory essentially considered the Coriolis force to be the force causing rivers to erode on one bank preferentially, thus initiating instabilities which could set up and perpetuate meandering. The effect has since been considered, in conjunction with the velocity profile of a river, to explain the genesis of secondary flow in rivers, and thus the genesis and continuation of meanders, (Scheidegger, 1970; Einstein, 1926). However, the effect has also been given less credit for the cause and maintenance of meanders by Chang (1969) who discounted the influence of the earth's rotation on the basis of a comparison of the cross-spectral analyses of depth and curvature data, using both "relative" (the positive and negative sign convention for curvature of flow) and "absolute" (all deviations assigned their positive or absolute values) curvature. Noting that the use of absolute curvature gave the higher correlations with other parameters, Chang assumed that the sign of curvature had no physical significance with respect to width, depth, or cross-section.

As the theory of transverse circulation, or secondary flow, developed, new schools of thought on meandering grew and gained acceptance. Matthes (1941) and Friedkin (1945) typified one school with the consideration that irregularities in bank erosion and sediment deposition constitute sufficient explanation for the meandering phenomenon. It was proposed that rivers flowing through erodable materials systematically transfer bank material to downstream reaches while eroding upstream banks, and the result of the erosion-deposition process is the gradual downstream progression of the entire river pattern. Valley-slope, bed-

load, discharge, rate of erosion of banks, bed-resistance, and transverse oscillations (not secondary flow, but seiche or gravity waves) were presented as the hydraulic variables interacting intricately to produce the migration. The actual physical interactions were not described mathematically, however, and the influential role of secondary flow was ignored, (Friedkin, 1945), or rejected emphatically (Matthes, 1941) by the erosion-deposition proponents.

It has since been emphasized that even in the absence of bank materials, streamflow often easily assumes a sinuous path. Supra-glacial streams that melt rather than erode and deposit their bank materials (Knighton, 1970) and even streamflow generated and maintained simply by water density differences, best exemplified by the Gulf Stream (Leopold, Wolman, and Miller, 1964), have exhibited the classical sinuosities that rivers flowing through erodable channels seem to imitate.

Seiche effects were explored further by Hjulstrom (1949). River turbulence was introduced as a likely hydraulic component of meander genesis, but little turbulence theory was available to provide adequate explanation. The seiche effect was given as a component of turbulence, while secondary flow escaped mention.

The use of flumes and hydraulic laboratories led Mockmore (1944), Shukry (1950), and Prus-Chacinski (1954) to revive and propound the notion of the influence of secondary currents in streamflow on river morphology. Spiral motion was observed in straight flumes as well as in curved flumes, and was connected with the observation of movement of bedload both downstream and laterally, towards the inside of bends. Helical flow satisfactorily provided theory supporting a differentiation in the magnitude of longitudinal and transverse tangential stresses on

the river bed and thus theoretically accounted for both the scouring and depositional actions of the river.

Still, all these classical explanations of sinuosity, proposing that plan form meandering is induced by the earth's rotation, regularities in erosion and deposition, seiche effects, and secondary currents, failed to allow the prediction of river geometry. Attempts to calculate meander forms on the basis of these models have met with minimal success. The processes definitely participate, in varying degrees, in the maintenance of the sinuous habit of a river, but none sufficiently defines the initiation or persistence of stream meandering.

C. Form Analysis

Another approach towards the understanding of the mechanisms of meandering entails the examination of river geometry and the building of models on assumptions of form. Initial attempts assumed meander loops to approximate segments of circles. Chatley (1940), Mockmore (1944), Shukry (1950), and even Prus-Chacinski (1954) postulated that streamflow followed paths constructed of combinations of circle segments. Ippen and Drinker (1962) also used arc combinations, but admitted the possibility of short, straight reaches as well, in order to better duplicate laboratory work. Yen (1967) considered a meandering channel with circular bends connected by relatively short tangents in order to explain spiral flow in natural rivers.

A more successful model for planform of streamflow is the sinusoid, postulated by Leopold et al (1960), Kondrat'ev (1962), and Toebe and Sooky (1967). Langbein and Leopold (1966) further proposed that the thalweg path approximates a sine-generated curve such that

river curvature is a harmonic function of a distance along the thalweg. The model is superior since it allows a random element to enter the determination of form. The development of the model assumes the curvature elements to be normally distributed and to proceed in the fashion of von Schelling's random walk (1951). The sinusoid smooths the model of the river by minimizing angular deviations in the channel curvature. At the same time, horizons for analysis of form are expanded through this model. Fourier analysis can be applied to the sinusoidal geometry to obtain form generalizations not previously possible with other models.

The analysis of visible planform properties of natural rivers constitutes the basis of an alternate approach taken towards the development of theory in fluvial geomorphology. Bates (1939), Inglis (1949), and Leopold and Wolman (1957, 1960) studied relationships among meander wavelength, meander width, and channel width. From these and similar studies, power functions using these variables have been formulated. Quantification of planform was more empirical and at the same time no less subjective than ever before in this approach. "Typical" sections of river were selected for study and definitions of the wavelength then depended upon the arbitrary choice of end points of the typical bend, (Langbein and Leopold, 1966). Even so, the separate analyses of process and form were useful in that they provided the natural and necessary background for the inspection of process-form interactions.

D. Process-Form Interplay

Jefferson (1902) presented an initial attempt to relate meander dimensions to hydraulic variables. He measured mb, the maximum width

of the "meander belt" for a number of rivers, and by listing the corresponding "md", or discharge, and the corresponding "w" or channel width, he concluded that the width of the meander belt is a function of stream width, and the lateral and vertical cutting of meanders is most intense at flood stages. Although his study was subjective both in its choice of reaches, and in the subsequent measurement of meander belts, his conceptual contribution to the study of the interaction of form and process in meandering was important as well as innovative.

Subsequent studies of geometric-hydraulic relations developed and improved upon Jefferson's ideas and methods. Friedkin (1945) explored the physical influences of slope on various aspects of the meandering form. He found that increased slope caused increases in meander width and wavelength.

Inglis (1949) rigorously extended the physical characterization of streamflow and found that meander dimensions generally seemed dependent upon the square root of the discharge. Leopold, Wolman, and Miller (1964) also accepted the square-root-of-discharge relationships as fact.

Leopold and Wolman (1957) studied the interaction of variables in laboratory rivers and concluded that the strongest relationships occurred between width and discharge. Discharge was demonstrated to be clearly influential in the modification of channel width, and width subsequently tended to control changes in meander wavelength.

Carlston (1965) studied natural rivers to investigate their hydraulic relations. He admitted that a major shortcoming of much of the fluvial work conducted (including the previous work of others as well as his own) was the subjectivity inescapable in the choice of

reaches examined. Often, workers based their findings on the measurement of a single meander rather than on a statistically valid sampling of meanders, and choice of workspace depended too highly on the availability of appropriate data, which was scarce. Carlston contributed a new concept to the theory of meandering: the dominant controlling discharges in the modification of meander wavelength are those ranging between the highest monthly discharge for a given year and the mean annual discharge. This conclusion was offered to replace Inglis' notion that discharges slightly greater than bankfull flow were the dominant modifiers of river meander wavelength (Carlston, 1965).

The proponents of this form-process interplay approach to the investigation of meandering generally agreed that linear planform parameters such as wavelength meander belt width, channel width, and wetter perimeter, depended directly upon the square root of the river's dominant discharge. Two distinct definitions of dominant discharge were involved (Inglis, 1949; Carlston, 1965). Slope was also accepted as influential on the basis of Friedkin's 1945 lab studies at Vicksburg. The proponents succeeded in their attempts to bring rigor into the discipline. However, fluvialogists generally examined planform on the basis of downvalley distance and width of meander belt, ignoring actual stream path-length. Meander belt was measured as the transverse distance between tangents to two adjacent or appropriate but opposite river bends, and it was used to describe the magnitude of the stream's oscillatory activities. Wavelength was approximated as the downvalley distance characteristic of the river's meandering wave. The path-length was not yet given any significant theoretical importance.

E. Sinuosity Considerations

Mackin (1956) and Schumm (1963, 1967) were among the first to examine the role of channel pattern in the form-process interaction. Mackin (1956) found the relation between sinuosity and cross-section to be such that as sinuosity decreases from tortuous to straight channels, the width-depth ratio of the channel increases. Sinuosity was then defined as the ratio of the path-length of the stream to the down-valley distance, a parameter still necessitating the inclusion of subjectivity into theory, because for all but perfectly regular meanders, the end points of the down-valley line must be selected arbitrarily.

Schumm (1963, 1967) compared the same sinuosity parameter to parameters of bank cohesion for several American and Australian rivers. He found that greater sinuosity values tended to characterize streams with more cohesive bank materials. Schumm defined cohesiveness by the percentage of silt and clay found within the bank material.

Considerations of stream sinuosity opened a new theoretical dimension in fluvial geomorphology. Different river patterns could be investigated to help strengthen previous notions and create new theory, but the elimination of subjectivity in the parametric characterization of sinuosity remained a problem of paramount importance for fluvialogists.

F. Contributions of Leopold: The Sine Generated Curve

Leopold and Wolman (1957) introduced a new and substantially superior definition of sinuosity when they considered it the ratio of the thalweg path distance to the valley length. As the thalweg is the river's own "preferred path", in that the thalweg will actually be the

river's course at very low discharges, the new sinuosity parameter described the river's meandering activities much more accurately. This concept was closely related to another significant discovery: the thalweg of a straight channel exhibits a sinuous tendency as it flows between pools and through riffles. With some laboratory experimentation, Leopold and Wolman (1957) demonstrated that a straight channel did not necessarily, or in fact usually, possess either a uniform stream bed or a straight thalweg.

The model thalweg inferred from the laboratory work had a wavelength of $L \times 2\pi$ radians (one complete sine wave) and was characterized by relatively deep pools at bends with shallows at points of inflection. This was offered, then, as a finding common to streams of both sinuous and straight channels. The wavelength was generally a function of width and so indirectly related to discharge. Bankfull discharge was assumed the best flow parameter in deriving flow-planform relationships. (Leopold and Wolman, 1957)

Leopold, Wolman, Miller, and Brush (1960) conducted laboratory experiments that produced beautifully sinuous channels containing no straight sections. They explained the meandering tendency as an energy dissipation technique. If bed material was of insufficient size to cause bed stabilization, the river would be free to create random bank projections. Local chance deflections would set up systems of eddies and vortices which would strain and erode the banks in the streams process of effecting internal energy losses.

Both sinuosity and energy considerations were greatly expanded in Leopold and Langbein's 1966 classic work, "River Meanders". Random, or stochastic, processes were finally allowed in meander development

theory. The random model used was based on von Schelling's (1951) random walk in which successive moves on a surface were independent with respect to direction travelled each move, but were governed by a probability function such that, relative to downvalley direction, the expected directional deviation at a point was a sine function of the channel distance travelled.

Streamflow was likened to the chance-directed particle which should supposedly seek a path eventually minimizing variation in directional changes. The sine-generated curve was postulated to be the flow pattern best allowing such minimum variance (given symmetrically curving paths of equal length), thus minimizing total erosion, as it was shown that banks are eroded at a rate directly dependent upon channel curvature. The sine-generated meandering was assumed, on the basis of field work, to provide the most uniform water surface, signifying maximum uniformity in energy expenditure per unit of distance along the channel. Thus, the sine-generated curve model was shown to eliminate concentrations of energy expenditure, following Newton's second Law of Thermodynamics. (Leopold and Langbein, 1962; Scheidegger, 1967)

Leopold still considered wavelength as a downvalley distance, however, and thus retained the subjectivity of previous analyses of form. A method that would eliminate this subjectivity, recurrent in hydraulic analyses of stream activity, had yet to be found.

G. Speight's Meander Spectra

MacKay (1963) initially proposed the application of the spectrum to studies of fluvial meandering, but he did not apply the technique himself. Speight's work initiated the actual application of the

technique.

Speight (1965) sought to overcome three significant biases in fluvial research. He complained first of the subjectivity involved as workers chose "typical" meanders on which to base wavelength statistics, and then of the error inherent in measuring wavelength along straight lines that probably have little relevance to actual flow direction. He finally concerned his research with the tendency of previous authors to ignore the possibilities of multiple wavelengths in stream form.

Speight used sets of air photographs of the Angabunga River in Papua in his research on curvature. He calculated successive angular deviations at points measured along the thalweg, separated by a constant interval along the path. He then applied spectral analysis, a technique described in detail in Chapter 2, to the derived series. The technique is used to separate complicated sinuous series into Fourier components assigning each Fourier term its variance contribution within the whole meandering series. Speight used the variance statistics as objective parameters of sinuosity. Multiple wavelengths were described for the first time through the use of this technique.

Attempts to relate the significant spectral peaks, or wavelengths, to tree-to-tree and grass-to-grass river widths were largely unsuccessful. The shape of the derived spectrum seemed to suggest the influence of channel cross-section, though, and this observation again opened new dimensions in fluvial work. Speight then attempted to compare spectral analyses on curvatures derived from the same river but from different years of flow activity. The air photos provided 24 years of the necessary data. No orderly meander migration was observed

but meander wavelength seemed constant. Spectral analyses was shown to have significant potential for the analyses of many other hydraulic variables, and Speight encouraged readers to expand research with this tool. (Speight, 1965).

H. Later Uses of Spectral Analysis

Thakur and Scheidegger (1970) continued the use of stochastic models in the study of sinuous river behavior. They considered a river course to be a stochastic line subject to certain geologic conditions. Spectral analysis was used to examine the curvature series of the 417 chain models they generated in an attempt to discover an expected or most probable spectral value. Although the expected value notion later proved less significant than they had hoped, Thakur and Scheidegger were able to conclude that the frequency distributions of deviation angles was reasonably Gaussian, an assumption of spectral analysis which had been previously unproved. They also developed the use of transition probability matrices for the determination of the mode or pattern of river bends, which was reflected in changes of sign in the series of the deviation angles.

Scheidegger (1970) in Theoretical Geomorphology, cites spectral analyses as an important tool in the measure of "wiggleness" of natural lines. The substantial potential for spectral analysis in turbulence studies is also recognized in his work.

Spectral analyses of sand waves has been very successful and shows great potential for the quantification of descriptions of bed roughness. Nordin and Algert (1966) were able to distinguish the spectra of sand dunes within long records, and they proposed a second-

order Markov process for wave formation. Ashida and Tanaka (1967) further developed theory on the second-order Markov model, and they used spectral analyses in the determination of the propagation velocity of sand waves. O'Loughlin and Squarer (1967) and Squarer (1968) recommended the use of spectral analysis to standardize descriptions of sand and waves. Nordin (1971) conducted flume studies of sand waves and concluded that the spectra of the waves were similar regardless of scale of phenomena. The general shapes and movements of both dunes and ripples in large of small flumes showed similar characteristic profiles, exhibiting only scale effects, when present, to provide differentiation. Nordin's findings agree reasonably well with Yalin's work of the same year. (Nordin, 1971; Yalin, 1971)

Chang and Toebe (1970) extended the use of spectral analysis of river curvature to the examination of absolute curvature of meanders. For this they used only absolute values of angular deviations along the Wabash and White Rivers in Indiana. They recommended the use of absolute curvature in cross-spectral analysis, although objective reasons for this were not clear. Chang found that the use of absolute curvature simply allowed higher correlations with the other parameters studied, and for this reason, he chose it as the superior statistic. Meander planform was demonstrated to be controlled by geology and soil, and flow rate and channel slope, through the use of contour maps of spectral amplitude statistics for overlapping successive downstream reaches.

This method, the "moving spectrum", is a way of avoiding serious non-stationarity in their data. Chang and Toebe were able to differentiate glaciated from non-glaciated fluvial landscapes on the basis

of their spectral findings. Meander wavelengths in unglaciated bedrock tended to be longer than those of glaciated regions.

Chang (1969) extended previous work, using the White and Wabash Rivers, to include cross-spectral examinations of width-depth, depth-cross-sectional area, width-cross-sectional area, curvature-width, curvature-depth, and curvature-cross-sectional area. Much of his work corroborated earlier theories, such as those relating planform parameters to the square-root of discharge, and others observing an increase in wavelength in the downstream direction. The analyses of Chang (1969) and Chang and Toebes (1970) constitute a long anticipated advance in objectivity and rigor of analysis not attained in earlier studies conducted by other fluvialogists. However, a variety of channel types must yet be examined, and comparative studies must still be conducted in order to test the findings of Chang's work based on the Wabash and White Rivers.

I. Other Recent Theoretical Contributions

Church (1972) applied various spectral analyses to sandur rivers on Baffin Island. He found a dominant pattern of peaks to be consistent, recurring at short, intermediate, and long wavelengths. Church found dominant wavelengths, expressed in terms of channel width, to persist as multiples of π .

Yalin (1970) used theories of river turbulence to explain the occurrence of sand waves and the regularities of meanders. Yalin's work predicted dominant meander wavelengths, expressed as multiples of mean channel width, to occur as integer multiples of 2π , and dune wavelength to be similarly proportional to $2\pi h$, where h is the mean water depth.

Further, Yalin has made a most important observation: that,

a systematic disturbance caused in the behavior of the largest macroturbulent eddies by a permanent discontinuity must be accompanied by the same and the reverse kind of disturbances occurring alternately, with equal length intervals, along the direction of flow. The dunes and meanders are the developed versions of the alternate erosions and accretions caused by these disturbances in the velocity field, and thus in the transport capacity of the flow.

(Yalin, 1970)

Surkan and VanKan (1969) extended work on random walk models, incorporating hydraulic considerations. They found existing models to be insufficient, in that simulated paths lacked the regularity of oscillation and freedom of erratic behavior found in nature. The inclusion of a gradient bias in random walk models improved models of meander patterns substantially.

J. Conclusions

Spectral analyses of both planform and hydraulic characteristics of meandering rivers provide objectivity and rigor that previous analyses lacked. The stochastic approach to river behavior provides a superior form of analysis at this stage in the theoretical development of the meandering intrigue. Stochastic models facilitate an understanding of process and form in fluvial geomorphology.

CHAPTER II

STOCHASTIC PROCESSES AND THE VARIANCE SPECTRUM

The major classical and contemporary explanations of the initiation and persistence of fluvial meandering were briefly explored in the previous chapter. Among the processes considered to be meander-generating were activities of secondary currents in the cross-sectional plane of a stream, influences of the earth's rotation, "regularities in bank erosion and subsequent deposition of materials, and seiche effects analogous to lake seiches. Attempts to calculate and predict meander forms and behavior on the basis of these models have met with minimal success. The processes examined definitely contribute to the establishment of the meandering habit of rivers, but none sufficiently explains either the genesis or geometry of meandering.

A. STOCHASTIC MODELS

Davis' (1893) firmly established mode of landscape analysis emphasized development according to a well-determined progression of stages. This leading role assigned to historical context in the examination of developing landforms was conceptually significant in that it allowed sensible development of theory on a macro-scale. Valuable generalizations were produced and some processes were isolated for further consideration. But studies made on a more micro-scale, examining detail within such a deterministic framework of logic, were complicated by the characteristic interaction of a large number of

inadequately understood variables. Fluvial behavior proved too dynamic for such "rigid" analysis of stages. Details of interactions were not satisfactorily understood using this "closed-system" approach.

The notion of landform equilibrium was inherent in the Davisian Stage analysis of landscape development. Other disciplines, such as biology and physics, commonly examined equilibrium interactions of ensemble variables in terms of systems theory. Chorley (1962) discussed the value of the application of systems theory to work in fluvial geomorphology. While the work of Davis and others in fluvial geomorphology was influenced somewhat by closed system logic, Strahler (1950) assumed that natural phenomena operated in both closed and open system fashions. Chorley (1962) recognized Strahler's work as a great theoretical advance for the discipline and he encouraged the development of theory on the assumption that natural systems are open.

In systems analysis, a closed system is an assemblage of objects and their relationships. Boundaries across which no energy export or import occurs are required in the definition of a closed system. Changes in the system are deterministic, progressing towards the establishment of equilibrium conditions. The progression towards equilibrium in a closed system simultaneously entails the maximization of entropy within the system. Thus, potential for change gradually decreases towards system stabilization. In this way, assumptions provide that initial conditions of the system are sufficient to determine the eventual equilibrium situation. While the deterministic model contains much useful information, it lacks the flexibility attained when systems are considered to be open.

Open systems assume assemblages to be characterized by dynamic

energy inputs and outputs, thus including closed systems as the special case of zero energy exchange. The theory of the open system's steady state allows more flexibility than the closed system's idea of equilibrium. Dynamic equilibrium, the steady state of open systems, is the dynamic condition in which the open system continually adjusts to externally induced physical changes by changing its own form. The concept of adjustment allows a more detailed analysis of form-process relationships than previously possible, and it admits a more liberal view of changes. The open system more easily directs attention to an entire landscape assemblage as it acknowledges the complex multivariate structure of most natural phenomena. It is thus a more realistic model for the study of river systems.

The most obvious advantage of the open system approach is its scope for the consideration of natural new inputs and their effects on the system. Random processes are thus allowed as stimuli, and responses to these are considered important. The model that best explains the behavior of hydrologic systems must recognize that the processes involved are not entirely deterministic but can be characterized as stochastic.

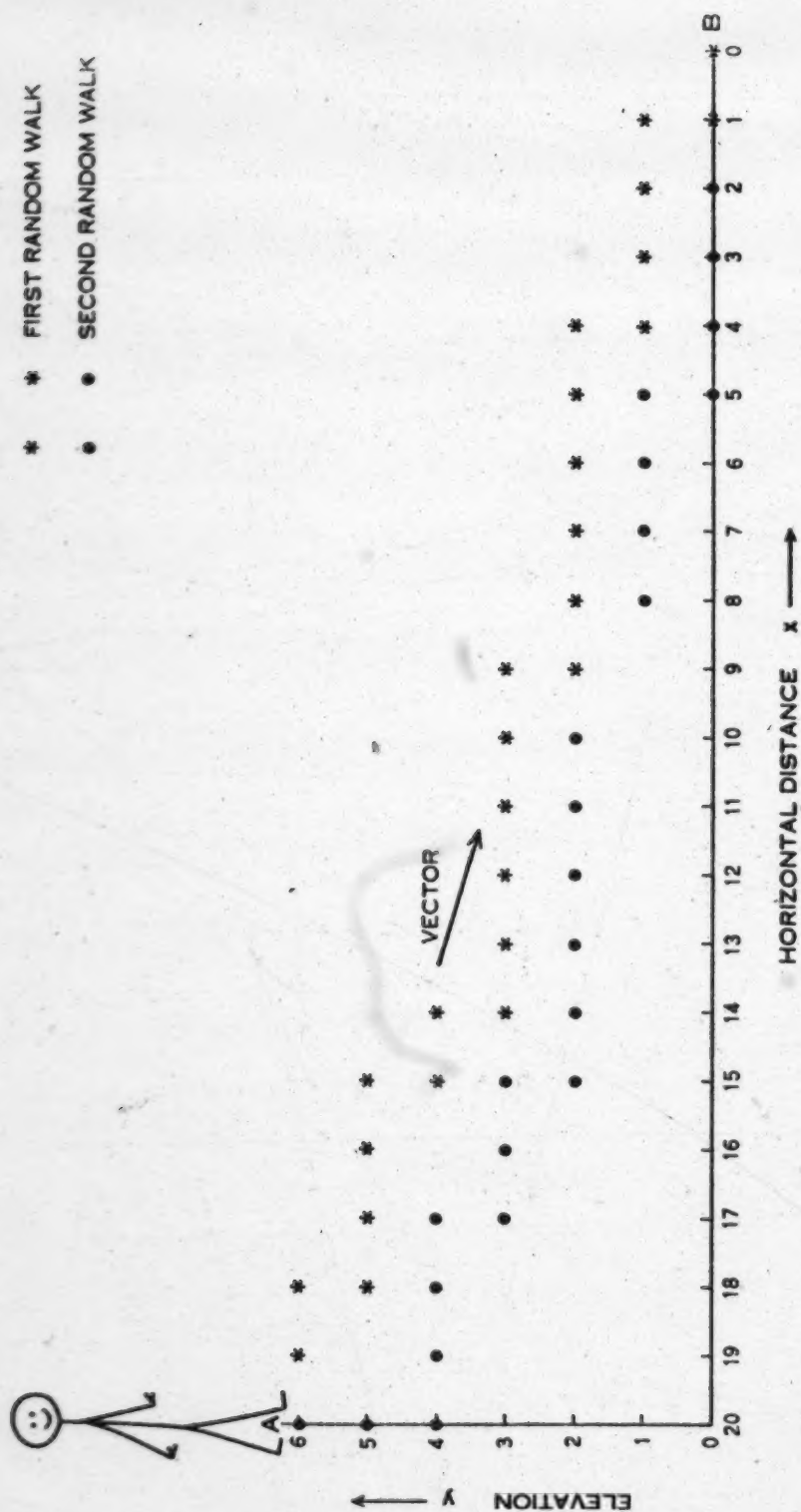
Precise descriptions of river morphology and behavior are valid for a river only at some specified time. The dynamic nature of fluvial processes is manifest over time as extensive changes are induced both gradually and catastrophically. Such changes become reflected in both the river's planform and bedform through adjustments by lateral planation, downcutting, or aggradation of the stream. The changes imposed are predominantly determined by the energy contained in the stream.

Fluvial processes must be considered as functions of both time and space. The analysis of river behavior with respect to such factors as discharge, slope, grain size of bed material, sediment load, or cross-

sectional form cannot be conducted independently of time and space considerations. Since flow constitutes a progression in both time and space, the factors studied will vary in such a way that they are independent of distant past or future changes, but dependent upon immediate past events, utilizing a portion of the information contained in an event of the immediate past. Connections with other points in time and space thus contain an element of determinism, but they also allow probabilistic relationships. Stochastic models provide for the co-operation of both deterministic and probabilistic processes, and they are thus the most suitable models to describe river behavior.

Figure 2.01 is useful in the clarification of the concept of a stochastic process. Suppose a man begins walking from a point A, intent upon reaching some point B six steps lower and twenty paces ahead. If he is permitted to move only forward or downward, then he has two choices each time he takes a step. Since he must reach point B, his decisions are influenced by his position relative to B, and probabilities may be assigned his choices. He has seven possible downward moves from which to choose (0, 1, 2, 3, 4, 5, or 6 steps down), and so for any position (x,y), his probability of choosing a downward step is $y/7$. Similarly, he chooses a horizontal step from position (x,y) with a probability of $x/21$. Probabilities of moves decrease as the man approaches B (or, as the x and y values decrease). The probabilities vary in a manner only partially determined by an determining last and next moves, and probabilistically independent of other past or future events. An element of influence can be associated only with time-adjacent moves. This type of model exemplifies the random walk and may be represented as

$$S_r = X_1 + X_2 + \dots + X_n$$



RANDOM WALK AS A STOCHASTIC MODEL - MAN MOVES FROM A TO B
 FIGURE 2.01

where S_r represents the position of the man taking a random step X_i at time t_r so that X_i indicates the distance covered in any one step. Here $\sum_{i=1}^N X_i$ may be used to describe the path distance between points A and B.

In the sequence of movements, the variable X_i at time t_r is independent of the entire previous set of X 's. The future cannot be completely predicted on the basis of the past record, but it relies both on the existing record and the probability of the next move. The probabilistic element precludes the use of deterministic models in recreating or simulating a series produced by the random walk model.

Von Schelling (1951, 1964) developed a variety of random walk models that were later applied to studies in fluvial geomorphology. Leopold and Langbein (1962, 1966) used the Von Schelling elliptical path model to explain their observation of the tendency of river meanders to assume geometrics minimizing the variance of the angular deviations along a path. Scheidegger (1967) further developed the model to extend his thermodynamic analogy between the movement of gas molecules and river behavior.

Stochastic processes may be used to explain the random nature of phenomena changing with time. The stochastic series may be represented as the family of random variables $X(t)$ where t , the time parameter, is an element of the set of intervals, T , or as: $[X(t), t \in T]$. The stochastic model accounts for both the probability and sequential relationship of the random variable. Conventional deterministic methods require the assumption that the process does not change with time or spatial variations, which is inherently contrary to hydrologic observations.

Any hydrologic time series may thus be represented as

$X_t = \epsilon_t \Delta t$, where X_t is the series, Δt is the deterministic component, and ϵ_t is the random or probabilistic element (Matalas, 1967). The uncertainty generally observed in the series is thus accounted for by ϵ_t , and this residual element is assumed always to be present. The deterministic element may not necessarily exist in the series, and when present, its importance in the prediction of X_t may vary from predominant to negligible in the determination of the process. The two elements may be mistaken for each other when records are very short, and it is essential that time series analysis minimize this possibility.

The amount of information retained or repeated by subsequent points in the time series may vary. Dependence between adjacent observations may be measured by the autocorrelation coefficient of the series. Markov chains are a special class of stochastic processes in which the value of X_r at any set of times t_r depend on the values X_s at any immediately previous set of times t_s . The outcome of any event is considered to depend on the outcomes of the immediately preceding events, but is independent of all previous events. A Markov chain "of order n" specifies that in a sequence of events, the outcome of each event depends on the outcomes of the n directly preceding event(s), but only on these. First order Markov processes are of particular hydrologic interest, as their patterns are recurrently found in river series. Adamowski and Smith (1972) found that the stochastic component of rainfall generation was adequately described by a first order Markov process. Markov processes without autocorrelation of series elements are described by the random walk model.

Any stochastic model by its very nature is not supposed to give

a perfect fit to the record it models. The actual record obtained is assumed to be only a sample of the many possible samples that could occur according to the probability distributions proscribed. An adequate model is rather one which preserves the desired parameters essential to the interpretation of the data. Markovian models preserve the lower order moments of a time-sequential distribution (mean, variance, and often skewness), and the first or higher order autocorrelation coefficients (Matalas, 1967; Rodriguez-Iturbe, Dawdy, and Garcia, 1971). The preservation of these parameters is generally adequate for work in fluvial geomorphology.

Stochastic processes are referred to as stationary when the statistics of the series are not affected by a shift in the absolute time or space origin, and so are reasonable approximations of the statistics of the appropriate ensemble. For instance, if the series is shifted K time units with respect to the origin, then, for the parameter x , $f(x; t) = f(x; t + k)$, must be true for all terms, K , under the assumption of strict stationarity of the series. A series is called weakly stationary, or stationary in the wide sense, if the expected values of the first and second moments remain constant when the time origin is shifted, and the autocorrelation (R_k) depends only on the time differences ($t_1 - t_2$) as:

$$E[X_{(t)}] = \mu_t = \text{a constant}$$

$$\text{and } \text{Cov}[X_{(t+k)}, X_{(t)}] = R_k$$

Stationarity is an assumption theoretically fundamental to higher order tests made on the time series, but it is generally assumed only in the wide sense, when considered at all.

In practise, rigorous tests of data stationarity are seldom run,

or, it run; seldom described. Tests for stationarity of data are not included in any of the literature to date applying spectral or cross-spectral analyses to the problems of fluvial meandering or bedform configuration. Tukey (1961) approached the problem more optimistically, encouraging a less-inhibited, while cautious, use of the spectrum:

. . . I have yet to meet anyone experienced in the analysis of time series data . . . who is over-concerned with stationarity. All of us give some thought to both possible and likely deviations from stationarity in planning how to collect or work up data, but no one of us will allow the possibility of non-stationarity to keep us from making estimates of an average spectrum . . . The fact that the spectrum is changing with time . . . need not make it unwise to estimate one, or several, average spectra Once we admit that we are estimating an average spectrum, we have admitted that there may well be other relevant characteristics of the situation beyond the spectrum, that estimation is not complete specification. Such an admission . . . is a good thing rather than a bad one. (Tukey, 1961)

It appears, then, that while stationarity should be considered in time series analysis, its absence should not alone prevent the use of spectral analytic techniques. Degree and form of stationarity of data should, however, be remembered in the interpretation of results.

Three main models of time series are used to simulate stochastic hydrologic processes. When the nonrandom portion of the series can be attributed to storage in the hydrologic system, simulation is best accomplished by the autoregression model, which may be represented as

$$x_t = f(x_{t-1}, x_{t-2}, \dots, x_{t-K}) + \epsilon_t$$

where x_t is the process under study and K is the time shift parameter.

(based on Chow and Kareliotis, 1970) A special case of the autoregression model is the linear autoregression model of the n th order:

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \dots + a_n x_{t-n} + \epsilon_t$$

where $a_1, a_2, \dots$ are the serial correlation coefficients of the variable x . When $n = 1$, the process defined is the first order Markov process:

$x_t = ax_{t-1} + \epsilon_t$. Figure 2.02 presents the correlogram for the first order Markov process, and Figure 2.03 shows the spectrum of this series.

Another special case of this model is the second order linear autoregressive scheme, in which two correlation terms arise:

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \epsilon_t$$

The scheme is similar to the first order process, but shows a slightly longer memory, such that events in the present rely on two events in the immediate past than on the preceding event alone.

A model useful in the representation of carry-over in a hydrologic system due to water retardation characteristics of the watershed is the moving average model. The model may be defined as:

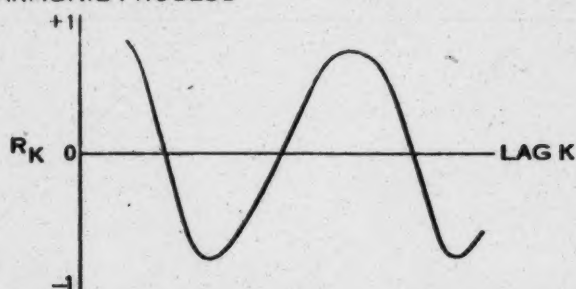
$$x_t = a_1 \epsilon_t + a_2 \epsilon_{t-1} + \dots + a_m \epsilon_{t-m+1}$$

where $a_1, a_2, \dots, a_m$ are the weights and m is the extent of the moving average. By virtue of the moving average on the ϵ 's, the model is not purely random, but is stochastic. (Chow and Kareliotis, 1970)

Oscillating variations in the sequence may be represented by the sum of harmonics model, where: $x_t = \sum_{i=1}^N (A_i \cos \frac{2\pi jt}{T} + B_i \sin \frac{2\pi jt}{T})$ where A_i and B_i are the amplitudes of oscillation, T is the period of cyclicity with $j = 1, 2, \dots$; and N is the number of record intervals. The model assumes the oscillations may be represented as sinusoidal. Diurnal, seasonal, and tidal periodicities in hydrologic systems may be approximated by the model. The model may also be applied to the meandering behavior of streamflow (Chow and Kareliotis, 1970).

The particular choice of model is not an obvious one, and may be aided by the use of the correlogram, as figure 2.02 shows. A correlogram

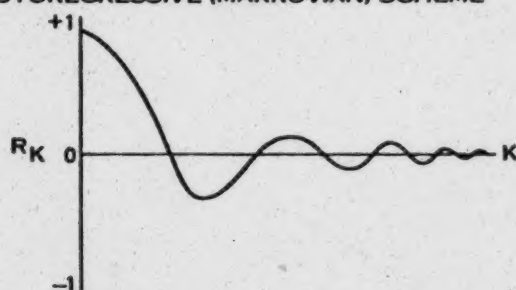
HARMONIC PROCESS



AN INFINITE SERIES OF PERIODIC CYCLES

PERIOD OF R IS NOT NECESSARILY THAT OF ORIGINAL SERIES

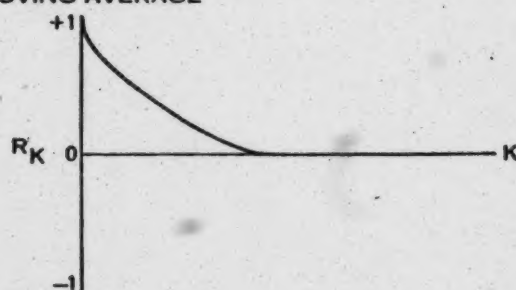
AUTOREGRESSIVE (MARKOVIAN) SCHEME



AN INFINITE SERIES GENERATED BY AN AUTOREGRESSION SCHEME

DOES NOT VANISH BEYOND ANY CERTAIN POINT

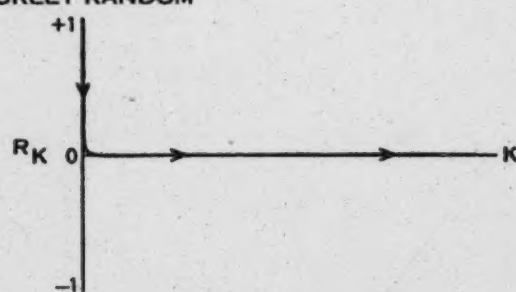
MOVING AVERAGE



AN INFINITE SERIES GENERATED BY A MOVING AVERAGE OF "M" TERMS

CORRELOGRAM TRACE IS A DECREASING FUNCTION TO "M" LAGS, ZERO BEYOND THE MTH LAG

PURELY RANDOM



CORRELOGRAM TRACE OF RANDOM* SERIES, SHOWS PERFECT CORRELATION TO ZERO LAGS, ZERO CORRELATION FOR ALL HIGHER LAGS

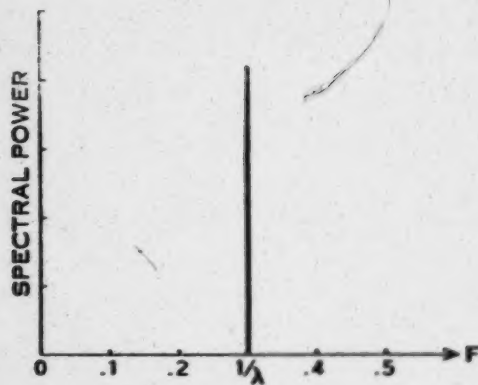
* MEMBERS OF SERIES INDEPENDENT OF ONE ANOTHER COMPLETELY

R_K SIGNIFIES THE CORRELATION COEFFICIENT FOR ANY SPECIFIED LAG "K" TIME UNITS

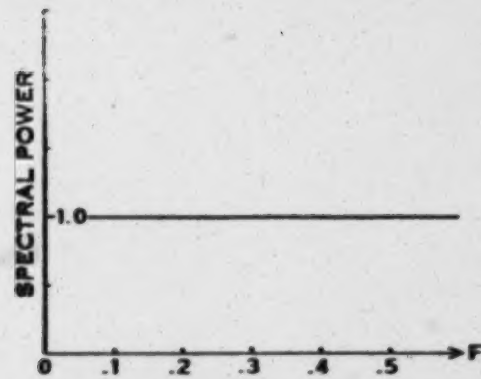
CORRELOGRAMS OF TYPICAL SORTS OF TIME SERIES

(AFTER FRIEDMAN)

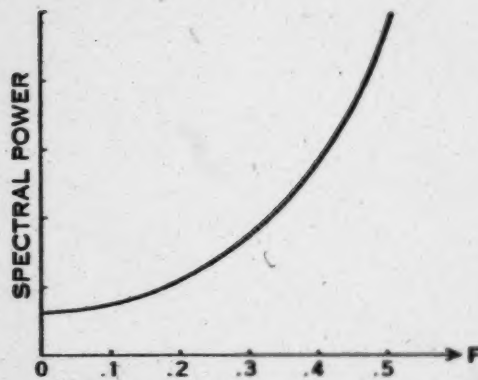
FIGURE 2.02



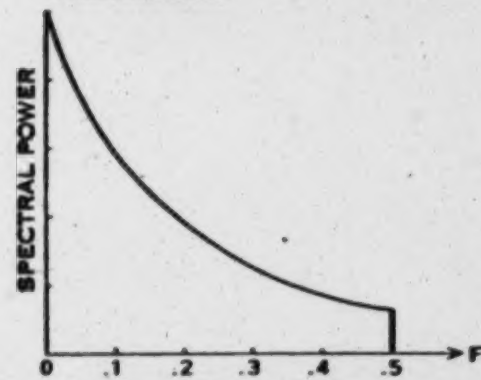
A) PERIODIC PROCESS,
SPECTRAL PEAK AT $1/\lambda$



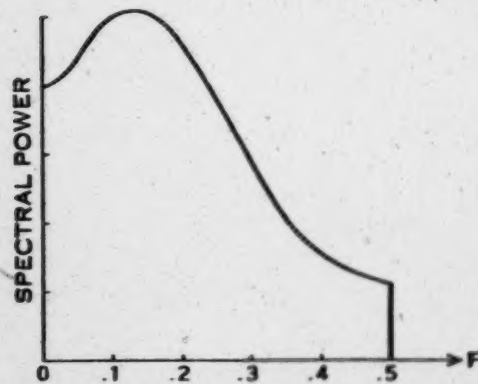
B) NORMAL WHITE NOISE (RANDOM),
POWER SPREAD ACROSS ALL
FREQUENCIES



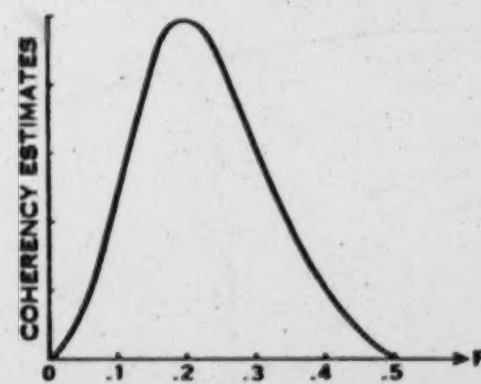
C) FIRST ORDER MARKOV PROCESS,
HIGH FREQUENCY CONTRIBUTIONS



D) FIRST ORDER MARKOV PROCESS,
LOW FREQUENCY CONTRIBUTIONS



E) SECOND ORDER MARKOV PROCESS,
(INTERMEDIATE FREQUENCY
CONTRIBUTIONS)



F) THEORETICAL COHERENCY
SPECTRUM FOR BIVARIATE
AUTOREGRESSIVE PROCESS

SPECTRA OF TYPICAL SORTS OF TIME SERIES (AFTER JENKINS AND WATTS)

FIGURE 2.03

diagrammatically shows the effect of a time origin shift on the series autocorrelation. The series is correlated with itself using a variety of lags, (denoted "m" for a series of "n" points) so that a comparison of the fit or correlation of the series to itself when shifted temporally or spatially may be obtained. The correlation is thus highest when the series are identical, as at a lag of zero units of time. The autocorrelation increases as the series approach identity. The correlogram presents the effect of each possible lag on the autocorrelation.

Every stochastic model includes G_c , the element of chance or probability. In any dynamic system, chance perturbations or fluctuations must occur and cannot be predicted. The element of uncertainty makes the use of deterministic models unsatisfactory for the study of hydrologic series. The elements of change over time and space in the behavior of fluvial systems are far too important to be ignored. The most reasonable approximation of the actual processes must be attained through the use of the stochastic assumption.

B. TIME SERIES ANALYSIS

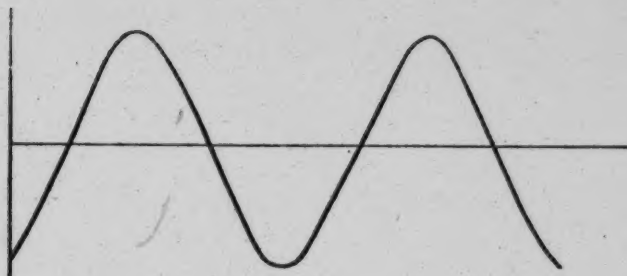
Recognition of the influence of time and space on series describing river behavior has led to the recent introduction of time series analysis into the discipline. Time series refers to a set of measurements recorded sequentially in time, with measurements usually sampled regularly from a continuous record (recorded at a constant time interval). Hydrologic series are constantly undergoing changes in time or space by natural or anthropogenic processes, but the changes may not be obvious in the actual record, or they may be difficult to detect

in short records where measurement errors may mask their presence.

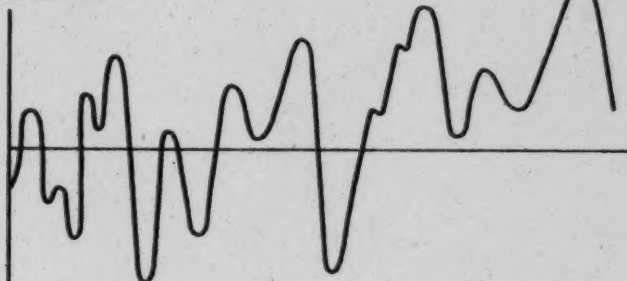
Haan (1972) recognized that one recurring problem in hydrologic studies is the lack of adequate data. The obvious solution to this problem is a systematic collection of long records of appropriate data. Since long series tend to contain information about the mean, and thus give better pictures of the processes under study (Matalas and Langbein, 1962), such a solution would seem imperative, but other problems are raised. If short records can mask the nature of the changes they contain, how much more confusing the interpretation of reams of continuous data will be! Methods for the analysis of long records of continuous time series are essential to a better understanding of the information they contain about both form and process.

The time series analysis model most suitable for the eventual explanation of meandering river behavior must be able to detect persistence in the data in all its forms of occurrence, and it must be sensitive to time dependence or serial correlation of the data. Meander and bedform data are likely to exhibit some combination of the four major components of time series. A periodic component, or one that exhibits regular reversals of curvature (approximating a sinusoidal form) is expected to characterize the data. A completely random component, or one showing no recurrent pattern, will likely exist within the data, as well, obscuring other process patterns. Many temporal and spatial series exhibit a long-term trend component so that the series changes systematically through time. Finally, the sample may encompass a major change in the series which is not really periodic; often incorporating some random fluctuations in period, phase, or amplitude; but is quasi-periodic or oscillatory in nature. Figure 2.04 illustrates these possibilities.

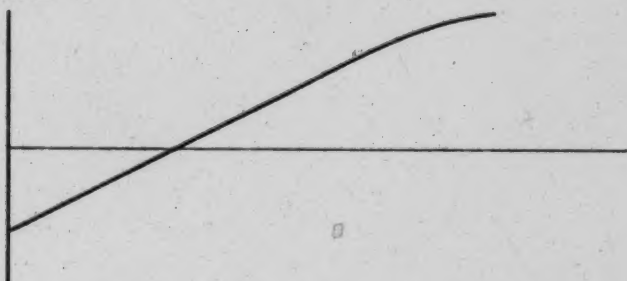
A) PERIODIC



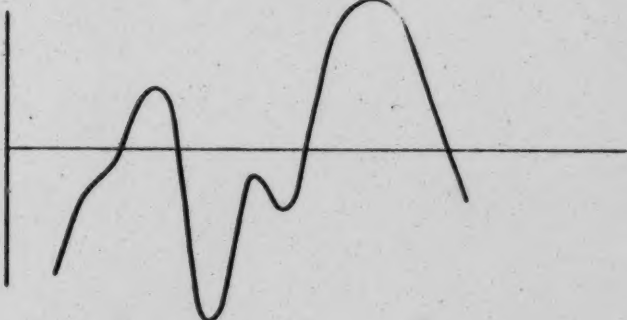
B) COMPLETELY RANDOM



C) LONG TERM TREND



D) QUASI-PERIODIC



COMPONENTS OF TIME SERIES
FIGURE 2.04

Each of the four possible components requires special treatment at some stage of the series analysis. A long term trend, if present, is usually removed at the beginning of the analysis by some sort of regression or moving-average process. Some forms of analysis instead consider a trend part of a low frequency oscillation, since "long-term" is a relative concept. A trend is usually assumed to be independent of the other components so that its removal does not affect any of the series' moments.

The general assumption of stationarity, mentioned above in section 2.A. and again in chapter 4, necessitates the removal of any trend present in the data. Stationarity in the wide sense requires the expected values of at least the first two moments of the series not to vary with origin displacement, while the autocorrelation function is required to depend only on time differences and not on absolute time. If higher order moments are similarly dependent only on time differences, the series is said to be stationary in the strict sense. If strict stationarity holds for all series in the appropriate ensemble, the series is further said to be ergodic. Available statistical methods are designed to analyze stationary series, often first assuming ergodicity. If the series cannot be assumed even to be weakly stationary, it should be transformed to a stationary series before rigorous analysis. Torelli and Chow (1972) give a detailed discussion of the problem of stationarity in hydrologic time series.

The most important aspects of time series, when the objective of study is discovery of the underlying physical mechanisms, are their oscillatory and periodic tendencies. Oscillations are commonly irregular in character, exhibiting erratic changes in amplitude, phase, and period

of the patterns. The major part of the theoretical work in time series analysis is concerned with obtaining an approximation of the variation in the series. This was originally approached through examination of the correlogram, but confidence limits and significance tests have never been successfully applied to the correlogram. For this reason, the technique was partially abandoned in favor of the analysis of the actual variance of the series. The correlogram is still a useful tool in the classification of processes underlying the series, but it is not desirable as the final product of the analysis since results cannot be easily compared in the absence of confidence limits or significance estimates.

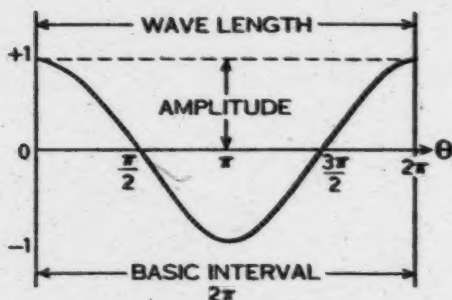
Periodic analyses commonly use sinusoidal models as shown in Figure 2.05, where:

$$y = \left(A \cos \frac{2\pi}{T} + \phi \right)$$

when A represents the wave amplitude, ϕ represents the phase angle or shift from the origin, and T represents the wavelength. The early assumption that time series were simple sums of a given wavelength led to harmonic analyses and later to periodogram analysis. The periodogram plotted periods present against their squared amplitudes. It was found to be a very unstable measure of periodicity of real time series, possibly due to random fluctuations in the data, and was then abandoned in favor of variance analysis.

The variation of a series may be calculated as a spectrum of contributions to the variance made by a variety of frequencies of oscillatory waves. The contribution to the spectrum by any frequency is called, by communication engineers, the power of the frequency in the series. The technique used to compute the spectrum of the time series is known

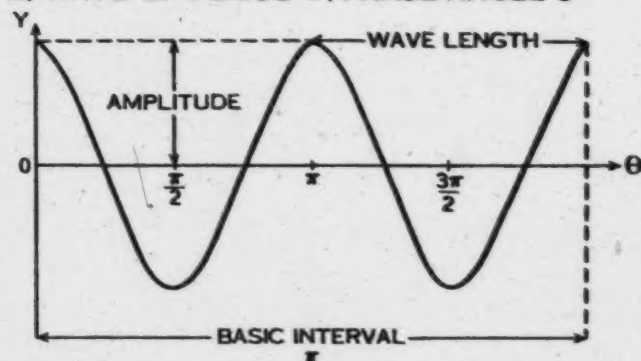
A) IDEAL COS WAVE, PERIOD 2π , PHASE ANGLE 0°



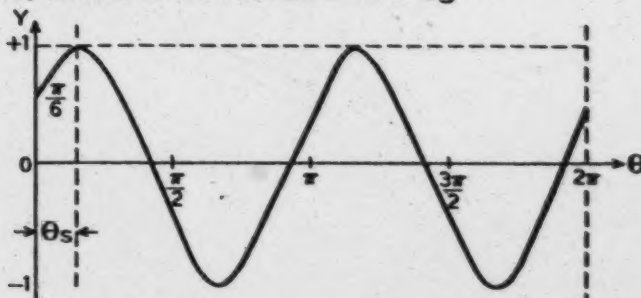
$$Y = \cos \theta$$

WHERE $0 \leq \theta \leq 2\pi$.
 θ IN RADIANS

B) WAVE OF PERIOD π , PHASE ANGLE 0°



C) WAVE WITH PHASE SHIFT θ_s



SINUSOIDAL MODELS FOR PERIODIC ANALYSES
 (AFTER RAYNER, 1971)
 FIGURE 2.05

as variance spectrum, or power spectral, analysis.

C. POWER SPECTRAL ANALYSIS

In general, spectral analysis is used to examine a data series for evidence of predominance of certain scales of activity or influence over others in the underlying physical processes. The technique is useful in the determination of some order from within the usually noisy recesses of an often large and complicated array of data. Basic concepts in the method of spectral analysis and some fundamentals of the mathematics of the technique are presented here as background to the use of the technique in this research. More curious readers are referred to the works cited for details of the mathematical processes involved in the calculation of the spectrum.

The notion of spectrum in spectral analysis implies the study of wavelength and period within time series. The spectrum estimates contributions made by both periodic and aperiodic functions within the stochastic series. It is initially assumed that the observed time series is a random sample of a continuous process which is composed of oscillations of all possible frequencies. A single series spectrum partitions the series variance into a number of bands or intervals of frequencies. The ultimate goal of spectral analysis is the determination of the nature of the generating processes such that predictions of future values, and occasionally so that simulations of historical records, can be achieved. A computation of the spectrum involves the calculation of the record mean, the autocorrelation function of the record, the Fourier cosine transform of each autocorrelation, and the estimation of spectral

density.

The formation of the autocorrelation function is the basic operation in spectral analysis. In this operation, the series is correlated with itself using a variety of lags so that a comparison of the fit of the series to itself with its time origin shifted is obtained. Most analyses find a lag ratio (time shift of m units within a series of n points) of 10 to 20 per cent to be optimal for the analysis. That is, $m/n < 20\%$ seems the best general estimator of characteristics of the series. A ten per cent lag is used in all of the analyses conducted in this study.

The measure of the autocorrelation is an ordinary linear coefficient varying from negative one (-1) to one (+1) through zero. The correlograms of Figure 2.02 again illustrate this method.

Harmonic analysis is based on the classical theories of Fourier Series, which postulate that a series of trigonometric generated curves, generally sine and cosine, may be used to represent any periodic and transient functions by a series of trigonometric terms--generally as sine and cosine waves. Harmonic analysis may be defined as the determination of a finite sum of sine and cosine terms present in the record, and their relative contributions to the variation in the series. .

The "harmonic" refers to the period observed when a specified portion of the data is considered. The fundamental or first harmonic has a period equal to the total length. The second harmonic will present one-half the period of the total, and the third will have one-third of the total period. Generally, the number of observations restricts the number of harmonics to $n/2$. Different harmonics are grouped or isolated according to frequency, and the importance of any particular frequency is then determined as its proportion of contribution to the total variance of the series.

The Fourier technique determines the harmonics by fitting sine and cosine curves, by a least squares method to the actual observed cycle. Generally, the first several harmonics describe the variation of the periodic function sufficiently well for purposes of interpretation of data patterns.

The Fourier series may be represented as

$$x[\theta] = \sum_{K=0}^{\infty} A[K] \cos(K\theta - \phi[K])$$
 where ϕ is the phase angle, or $\frac{360^\circ}{T}t$, for t representing any specific point selected from the total time T . The series expands as

$$a[0] \cos 0 + a[1] \cos \theta + \dots + a[K] \cos K\theta + \dots$$

$$b[0] \sin 0 + b[1] \sin \theta + \dots + b[K] \sin K\theta,$$

thus represented as $n/2$ cosine terms and $n/2$ sine terms (after Rayner, 1971).

The constants a and b are found as

$$a[K] = \frac{2}{n} \sum_{j=1}^{n-1} \left[x[j] \cos\left(\frac{360^\circ jK}{n}\right) \right]$$

and

$$b[K] = \frac{2}{n} \sum_{j=1}^{n-1} \left[x[j] \sin\left(\frac{360^\circ jK}{n}\right) \right].$$

The variation contributed to the series by any specific harmonic may be expressed as $\frac{C_K^2}{2}$, for $C_K = \sqrt{a_K^2 + b_K^2}$, and so C_K has the amplitude of the K th harmonic. The ratio of $\frac{C_K^2}{2}$ to the total variance explains the proportional contribution of the specific harmonic. Since the harmonics are independent and not correlated, no two harmonics can explain the same portion of the variance of X . Thus, contributions can be added so that if the first harmonic contributes 20 per cent of the total variance; the second, 50 per cent; and the third, 25 per cent, then all other harmonics contribute only five per cent.

The intensity of particular harmonics provides a measure of the

relative importance of each to the entire series variation. This may be shown in a periodogram, but the spectrum is a more versatile description of power since it decomposes aperiodic contributions, as well.

Chance fluctuations may enter the series through the mode of recording, and will influence conclusions according to the difficulty encountered in sampling techniques. Since these fluctuations could reduce the adequacy of the spectrum it may be desirable to minimize their effects. Various techniques or "filters" are available to amplify or reduce the apparent influence of certain frequencies within the data. Filtering is also used in detrending the original series.

High pass filters allow the retention of only high frequencies in the data. Low pass filters similarly allow only the lower frequencies to remain in the series. Averaging is a form of low pass filtering as it removes high and some medium scale disturbance. Two common filters in spectral analysis are the differencing filter and the moving average filter. The differencing filter allows medium to high frequencies to pass unaltered, while lower frequencies are substantially attenuated. The differencing process may be represented as:

$$Y_t = X_{(t)} - X_{(t-1)}$$

where $X_{(t)}$ is the value of the raw data at time t , and $X_{(t-1)}$ is the original value at the previous point in time, $t-1$. Y_t defines the residual data at position t . A gain, $G_{(f)}$, occurs in the residual analysis such that:

$$G_{(f)} = 2 |\sin \pi f|, \quad 0 \leq f \leq \frac{1}{2},$$

where f is the specified frequency given in cycles per Δt (after Chang, 1969). Moving average filters are similarly designed to pass high frequencies.

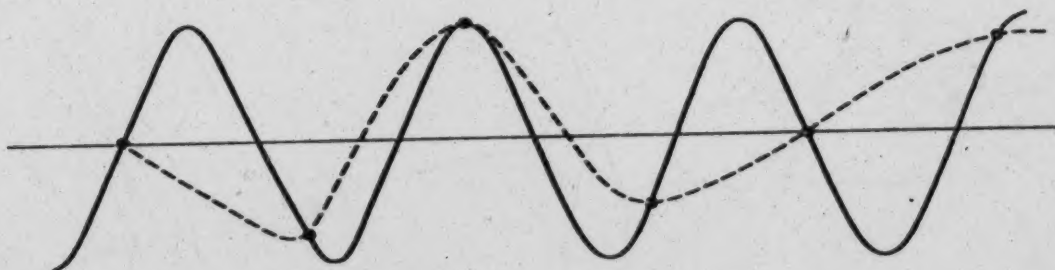
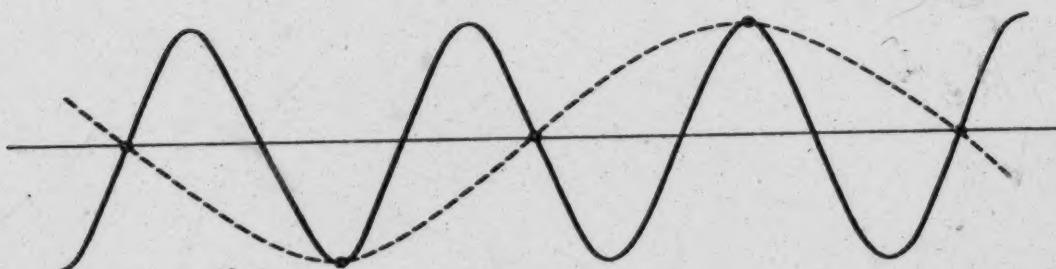
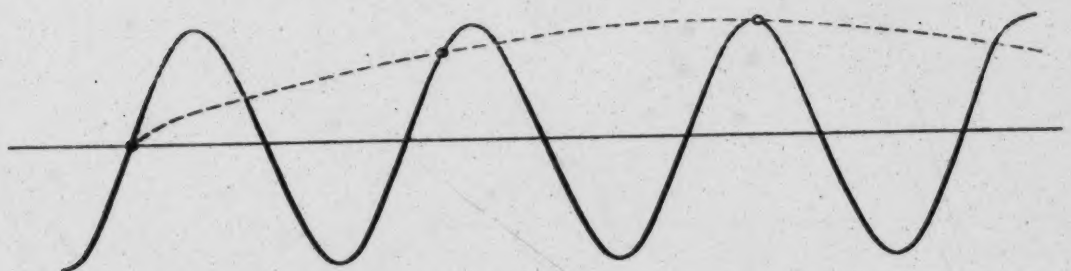
Design of sampling interval and subsequent analysis must try to prevent "aliasing" in the series. Figure 2.06 depicts the aliasing problem. The sampling interval equals or exceeds one half the period to be examined, and thus the true period is never recognized. Only when the period exceeds twice the sampling interval can the event be measured, and the record length necessary to determine the event increases as the period of the event approaches a value of twice the sampling interval. The frequency (f_n) which limits the events that can be determined by any sampling frequency (f_s) is commonly called the Nyquist or folding frequency, in which $f_n = f_s/2$. Sampling designs must consider this phenomenon carefully in order that all events considered plausible are allowed a reasonable chance of detection, if possible.

D. CROSS-SPECTRAL ANALYSIS

Cross-spectral analysis extends the use of the technique to the simultaneous consideration of two series. "Correlations" of similar frequency components between the two series are tested by the "coherence" statistic, which is analogous to the squared correlation coefficient in correlation analysis. A plot of coherence against frequency can be useful in the identification of patterns of correlation, and therefore, of dependence between the two series.

Cross-spectral analysis requires assumptions similar to those of its univariate analogue. Both series under consideration must be stationary, at least weakly, and random elements of each series should be Gaussian. Each series should be detrended before analysis.

The cross-covariance function is analogous to autocovariance, and



---- ALIASED RECORD
 • SAMPLING TIME
 — TRUE RECORD

THE PROBLEM OF ALIASING — OBSERVING PERIODIC
 EVENTS VIA SAMPLING INTERVALS OF LESS THAN
 ONE-HALF THE PERIOD (AFTER GUNNERSON)
 FIGURE 2.06

is similarly basic to the cross-spectrum. For two time functions $F_1(t)$ and $F_2(t)$, the cross-covariance is

$$R_{XY(m)} = \frac{1}{T} \int_{-T/2}^{T/2} F_1(t) F_2(t+m) dt,$$

or, in expected value notation, it is simply:

$$R_{XY(m)} = E [(X(t) - \mu_X)(Y(t) - \mu_Y)]$$

where m is the lag specification and μ , the series mean.

The value of $R_{XY(m)}$ is always a real number. Symmetry for the cross-covariance occurs as:

$$R_{XY(m)} = R_{YX(-m)}.$$

The cross-correlation function is obtained from the cross-covariance, and measures interactions between two processes. It may be represented as:

$$\rho_{XY(m)} = \frac{R_{XY(m)}}{\sqrt{R_{XX(0)} R_{YY(0)}}}$$

Its univariate analogue is the autocorrelation function.

The Fourier transform of the cross-correlation of two series gives the cross-spectral density function for the series. The cross-spectrum is the complex product of the real amplitude spectrum (cospectrum) and the imaginary phase spectrum (quadrature spectrum). The cross-spectrum may be represented as

$$P_{XY(\omega)} = 2 \int_0^{\infty} R_{XY(m)} \exp(-2\pi i \omega m) dm$$

with its components the cospectrum:

$$P_{XY(\omega)} = \int_0^{\infty} (R_{XY(m)} + R_{YX(m)}) \cos 2\pi \omega m dm$$

and the quadrature spectrum

$$Q_{XY}(\omega) = \int_0^{\infty} (R_{XY}(m) - R_{XX}(m)) \sin 2\pi \omega m dm.$$

The ratio $\frac{P_{XY}}{Q_{XY}}$ gives a measure of the phase relations between the two series, and $\arctan\left(\frac{P_{XY}(f)}{Q_{XY}(f)}\right)$ gives the phase angle $\phi(f)$. When coherence estimates are high or close to unity, the phase $\phi(f)$ approximates well the relationship of the phase between the two series analyzed. However, low coherence is an indication of a poor relationship of phase and so a poor correlation at the specified frequency. (see Edge and Liu, 1970)

Coherence is an analogue of correlation, but is given as a function of frequency as:

$$Coh = \frac{P_{XY}(\omega)}{\sqrt{P_{XX}(\omega) \cdot P_{YY}(\omega)}}$$

It measures the correlation of the series within a small range of frequencies. Again, more detailed outlines of cross-spectral analysis may be found in Rayner, (1971), Matalas, (1966), and especially Jenkins and Watts (1966).

E. APPLICATION

Spectral analysis is a valuable technique in the development of the theory of river behavior. Spectra of both meandering and depth patterns allow some interesting comparisons, leading to theory, regarding the generating processes. Cross-spectra of the two series provides valuable insight into a more encompassing set of processes.

Spectral analysis is extremely useful in the condensation of masses of continuous data into meaningful summaries of form, and is also important in that it can provide insight into generating processes. Its

use in studies of meandering is still in an exploratory stage, but it seems a promising technique, appropriate to the solution of many old problems.

Chapter I described some of the work previously done in the examination of planform and depth patterns in rivers, and reference is now made to those studies which have used spectral techniques.

Mackay (1963) first suggested the application of the spectrum to meanders, but Speight's (1965a, 1965b, 1965c) work was the first substantial applied contribution. Thakur and Scheidegger (1970), Chang and Toebes (1970), Yalin (1971), and Church (1972) all applied spectral techniques to meandering. Nordin (1971), Church (1970), Yalin (1971) and Chang and Toebes (1970) applied spectral methods to bed configuration downstream depth changes. The only work done so far utilizing cross-spectral analysis of meandering and depth is by Church (1972) and Chang (1969), although some of Yalin's theoretical predictions can easily be extended to cross-spectra. The findings of these works were summarized in chapter I. Observations based on the data analyzed in this study, relevant to these works, will be presented in later chapters.

CHAPTER III

COLLECTION OF DATA

A. DATA NECESSARY FOR THE PURPOSES OF THIS STUDY

The theory that the meander plan of rivers is best modelled by a sine-generated curve was formally proposed by Langbein and Leopold in 1966. The theory postulates that a strong sinusoidal relationship exists between angular deviations of streamflow and distance travelled along the curved path of flow. It was also hypothesized that for rivers characterized by both meandering and straight reaches, the meandering reaches should exhibit lower variances in certain of their hydraulic factors (depth, velocity, local slope; also the planimetric geometry of the river). Langbein and Leopold (1966) assumed that the meandering river simultaneously maintains greater changes in bed contours than the straight river, the factors in combination finally rendering the meandering plan the more stable flow pattern.

Meander planform has not commonly been examined over long reaches of streamflow. More usually, single meanders have been assumed to characterize the pattern (Leopold and Wolman, 1960; Langbein and Leopold, 1966), and thus single wavelengths have been chosen as sufficient description of plan morphology. Speight (1965) introduced both the study of long reaches of planform and the immediate observation that multiple wavelengths recurred to characterize the plan. Thakur and Scheidegger (1970) and Chang and Toebes (1970) have extended the analysis of planform as series of deviation angles, but data on changes of

directional angle of streamflow is scarce.

Accurate information detailing the longitudinal profiles of river depths has been similarly scarce. Chang (1969) analyzed thalweg depths of the Wabash and White Rivers in Indiana using depth information on maps. Low-flow depths mapped at non-equal intervals were used to construct a more continuous series of depth measurements through interpolation. Although it is possible to obtain continuous depth records of a river's longitudinal profile, no known depth analyses have yet exploited this potential.

The paucity of adequate plan and depth information to corroborate or at least test such important theories as those mentioned earlier (Langbein and Leopold, 1966) emphasizes the need for further field study oriented towards a stochastic examination of the behavior of rivers. Particularly, analyses based on continuous depth information should provide useful comparisons involving both previous studies and existing theories.

B. CHOICE OF RIVERS FOR THIS STUDY

Continuous depth records of longitudinal river profiles (at high stage) and series of angular deviations of planform were obtained for three local accessible rivers (Beaver, Alta.; Kootenay, B.C.; South Nahanni, N.W.T.). The general locations of the rivers are shown on page 51 in figure 3.01 (a map). The rivers were chosen for their similarities to and for their differences from those of previous studies by other authors, allowing the best comparisons possible within practical limits. Co-operation of interested federal and provincial agencies, availability of adequate maps and air

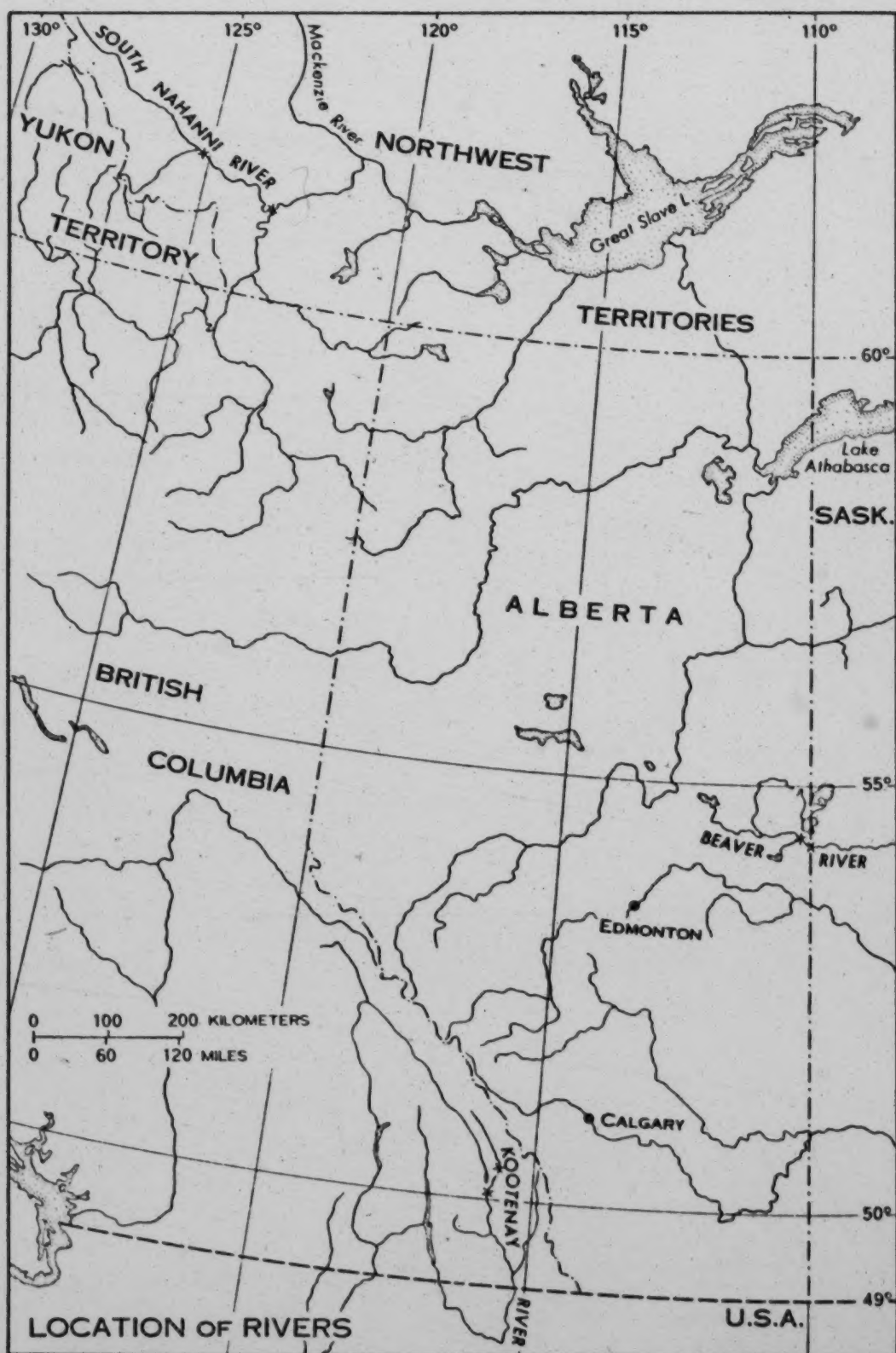


FIGURE 3.01

photographs, existence of stage records and discharge statistics, and actual accessibility of rivers constituted the practical constraints, but these criteria fortunately interfered little with the choices made.

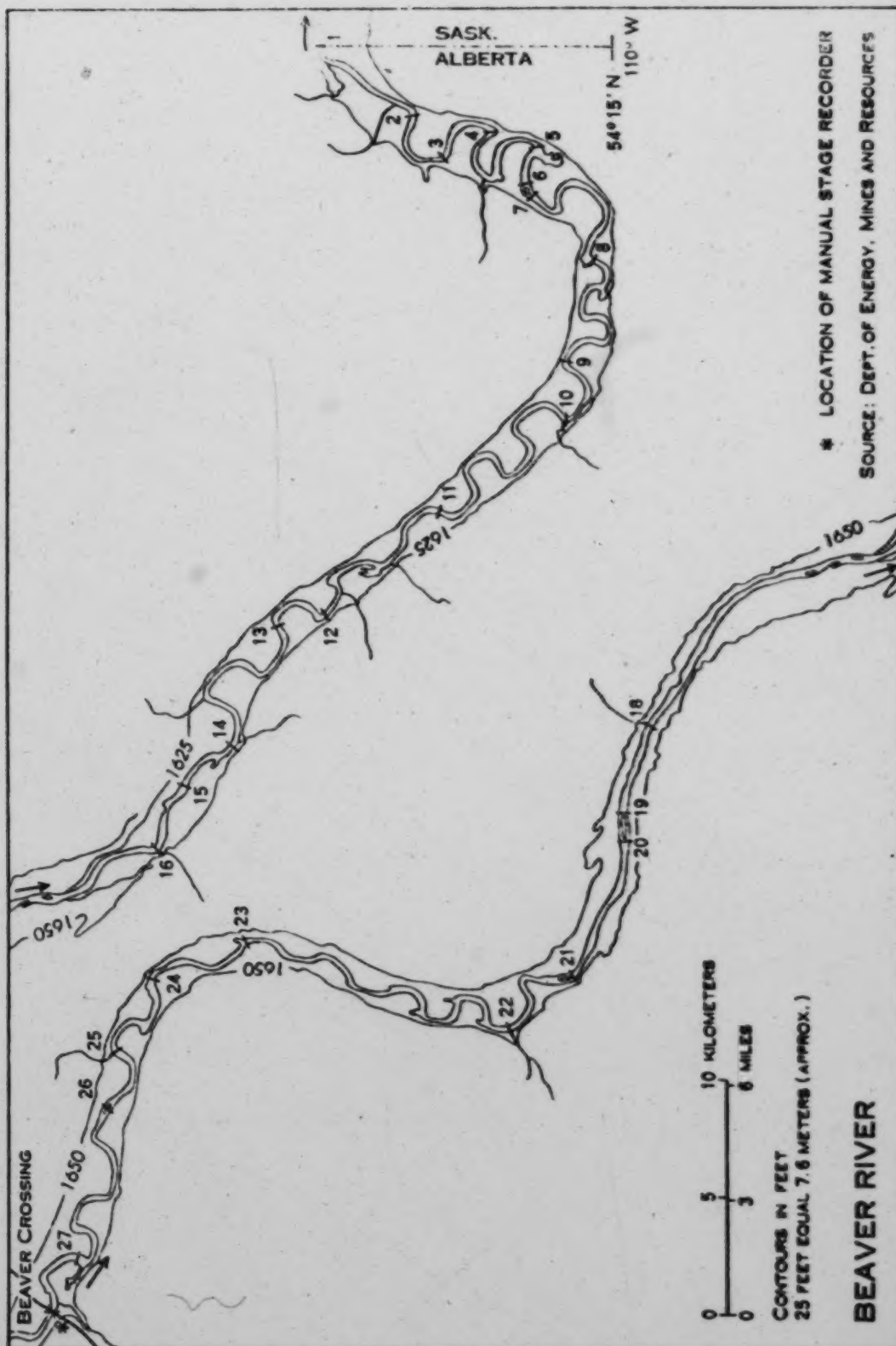
The Beaver River exhibits regular meanders (within a spillway valley, relic of the Pleistocene glaciation) as it flows over a sand bed, thus providing potential for comparison with the work of Chang and Toebes (1970) on the Wabash and White Rivers. The South Nahanni River meanders, deeply incised, over a gravel bed, through very old canyons that have escaped recent glaciation(s). This river thus provides an interesting analogy for work on the Colorado River (Leopold, 1969). The Kootenay River provides a useful contrast for these as it meanders irregularly, in some reaches entrenched, though not as deeply as the South Nahanni, and has a gravel bed. Detailed descriptions of these rivers will further emphasize their adequacies with respect to the purposes of this study.

1) The Beaver River

a) Geology:

A meandering valley, cut by a much larger river active during ice melt, today contains and restricts the smaller (misfit) Beaver River (Neill, 1963). The old valley served as a spillway for large meltwater lakes. Its meander belt is about nine hundred feet (275 meters). The Beaver maintains a course of regular meandering within this valley, as shown by the map on page 53 (Figure 3.02) and again by the air photo in Figure 3.06 on page 79.

The Beaver River flows over a bed of clean medium to coarse sand



* LOCATION OF MANUAL STAGE RECORDER
SOURCE: DEPT. OF ENERGY, MINES AND RESOURCES

BEAVER RIVER

FIGURE 3.02

and between banks that are generally silty but expose sand in places. Borings have shown the sand to continue for forty feet below the banks (Neill, 1963). The Beaver's average bankfull width of about 180 feet (or 55 meters) is maintained with little variation throughout the reach studied. Mean slope has been measured along the channel as approximately 0.24 per 1000, and this seems to vary little with stage (Neill, 1963).

An exceptional flood in 1962, during which the Beaver overtopped its banks by about ten feet, has been described as the Beaver's one-hundred year flood. The flood provided an excellent opportunity for process study, and Neill (1963), who was primarily concerned with the problem of pier scour, was able to observe some bedform evolution in the course of his study. A longitudinal sounding of a small portion of the river's midstream profile was obtained for near-bankfull stage. (At the time of the survey the Beaver was approximately one foot below bankfull stage and twelve feet below its flood peak). Two subsequent longitudinal soundings over a three-mile length of the same reach allowed comparative bedform examination.

Neill (1963) observed, from the soundings, that shallows and deeps which had become very exaggerated during the various stages of flood had adjusted to form a less extreme profile by the time the Beaver returned to its normal low stage. Bedforms of three distinct types, classified by size, were recorded. Long waves (crest to crest approximately three hundred feet or 90 meters), dunes (about ten feet or three meters between crests) and ripples (dimensions in inches or centimeters) were observed, but these were regarded only as interesting observations and were not the main concern of Neill's study.

Neill also recorded observations on some other flood effects.

Deposits of fresh sand were recorded on the insides of all sharp bends, while little bank erosion had been noticed. One bend had undergone considerable erosion on its right outer bank and simultaneous deposition on its left inner bank, resulting in a net conservation of river width (Neill, 1963). It is geomorphologically important to note both the occurrence of the flood and any possible resultant changes in planform (the only reference available notes that these were not substantial), because the maps on which planform data for this study are based were compiled from pre-flood air photographs, even though they were the most recent available.

Large boulders are exposed along the Beaver's banks wherever the river attempts to cut laterally into the old valley fill of a past glaciation. The modern Beaver does not possess the competence necessary to move these rocks, which thus serve to protect or armour the river's banks. It is very likely that the boulders passively helped conserve the Beaver's planform during the 1962 flood. This would explain why almost no changes in planform were observed (Neill, 1963) throughout the various stages of flood. Such an armouring effect of these boulders would ensure the adequacy of the maps used in obtaining planimetric data.

b) Vegetation:

Low forest and dense bush cover the valley flats where cultivation is not widespread. A number of new roads into the valley have recently been constructed, however, indicating a trend towards expansion of valley cultivation. At the time of the river survey conducted for this study, most of the land adjacent the river supported willow (Salix spp.),

aspen (Populus spp.), dogwood (Cornus stolonifera), and a variety of sedges and grasses.

c) Climate

Table 3.01 summarizes climatic data for the areas of the three rivers studied. Mean monthly precipitation and temperature figures for the Beaver indicate that spring runoff normally occurs during late April, continuing through May, while precipitation is heaviest through the summer months. Highest flow conditions are thus expected to characterize the stage of the Beaver through mid-to late May. The river will also flow at high stage (more sporadically) many times through the summer months (after storms). Neill et al (1970) have found that, normally, fourteen per cent of the Beaver's annual runoff occurs in April and another nineteen per cent occurs in May. Figures for runoff are similar for June and July, but in these months, runoff is likely more flashy. The survey of the Beaver was thus planned for mid-May.

d) Flow Information

The Beaver River flows northwest to join the Churchill River, eventually emptying into Hudson's Bay. It drains an area of 5460 square miles (14,100 km.²). A manual guage, read daily, has been used since October, 1955, to collect data on the flow of the Beaver River. Its location, near the Cold Lake Reserve, is marked on the map of Figure 3.02, on page 58.

The mean annual discharge of the Beaver River is about 936 cfs. (or $26.5 \text{ m}^3 \text{ s}^{-1}$). Extremes have occurred on June 13, 1962 (the flood peak mentioned earlier) when a maximum instantaneous discharge of 21,800 cfs. (or $617 \text{ m}^3 \text{ s}^{-1}$) was recorded, and on February 15, 1964, when a minimum daily

Table 3.01: NORMAL CLIMATIC DATA FOR RIVERS STUDIED

1. Precipitation
(in inches)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual Monthly total average
Beaver River	0.99	0.71	1.03	1.08	1.41	3.29	3.49	3.14	1.66	0.75	1.00	1.09	21.1
Kootenay R.	3.30	1.85	1.04	1.02	1.80	2.91	1.01	1.66	1.61	1.65	2.21	2.27	22.3
South Nahanni R.	0.68	0.47	0.57	0.68	1.13	1.42	2.14	1.88	1.28	0.73	0.96	0.88	12.8

2. Temperature
(in degrees F)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Average
Beaver R.	-2.1	7.5	18.0	35.5	50.1	56.6	62.8	58.8	50.0	39.8	21.9	11.8	34.2
Kootenay R.	19.5	24.1	34.6	45.3	54.2	57.2	66.1	62.9	55.2	44.2	30.3	24.2	43.2
South Nahanni R.	-15.8	-10.0	4.8	25.8	46.3	57.2	62.0	57.9	46.2	29.9	6.3	-10.9	25.0

*1 met. station at Cold Lake: 54°25'N, 110°17'W, 1784 feet a.s.l.

*2 met. station at Canal Flats Ranger Station: 50°09'N, 115°34'W, 2680 feet a.s.l.

*3 met. station at Fort Simpson: 61°52'N, 121°21'W, 432 feet a.s.l.

(source: data from the "Monthly Record - Meteorological Observations in Canada" - Environment Canada.)

A. REGIONAL CONTEXT OF THE RIVERS

River	Location of river; recorder	Drainage Basin	* Drainage Area, mi ² ; Km ²	Recorder in Reach	Contact
Beaver River	Alberta;	Churchill,	5460; 14,100	manual	Inland Waters Directorate, Calgary Office, Water Survey of Canada
(Cold Lake	54°21'20"N, 110°13'00"W	Western Hudson Bay			
South Nahanni River	NWT;	Mackenzie,	12,900; 33,313	recording	same as above
(Clausen Creek)	61°15'10"N, 124°02'10"W	Arctic Ocean			
Kootenay River	British Columbia;	Columbia,	2080; 5371	manual	Inland Waters Directorate, Pacific Region Water Survey of Canada
(Canal Flats)	50°08'52"N, 115°47'57"	Pacific Ocean			

* refers to area above recording station

Table 3.02: General Flow Information for the Rivers, part 1

B. FLOW DATA

River	Slope (per 1000)	Mean Annual Q cfs; $m^3 s^{-1}$	Extreme max. Q cfs; $m^3 s^{-1}$	Extreme min. Q cfs; $m^3 s^{-1}$	Mean annual flood cfs; $m^3 s^{-1}$	Mean Q ₃ for survey cfs; $m^3 s^{-1}$
Beaver River (Gold Lake)	0.24	936; 27	21800; 620	13.0; 0.4	5870; 166 (late May)	962; 27
South Nahanni River (Clausen Creek)	2.25	14,700; 416	290,000; 8210	1690; 48	62,000; 1755 (mid-June)	24,798; 702
Kootenay River (Canal Flats)	2.8	3,140; 89	29,700; 841	178; 5.0	19,000; 540 (early June)	10,900; 309

Table 3.02 continued: General Flow Information for the Rivers, part 2

discharge of 13.0 cfs. (or $0.4 \text{ m}^3 \text{ s}^{-1}$) was recorded. The mean annual flood flows at 5,870 cfs. (or $166 \text{ m}^3 \text{ s}^{-1}$). Sediment data is not available for the Beaver River.

During the time over which the survey was taken for this study (May 15, 1973), the mean discharge was approximately 962.0 cfs. (or $27.2 \text{ m}^3 \text{ s}^{-1}$), slightly in excess of the annual mean flow.

2) The South Nahanni River

a) Geology

Ford (1972) describes the South Nahanni Valley as the "most 'fluvial' landscape in Canada". The region (Ford, 1972) escaped the Wisconsin glaciation so that during the past 300,000 years, fluvial processes have been the foremost processes acting to modify the landscape. Isostatic rebound from the most recent glaciation, the "First Canyon Glaciation," has been responsible for the fluvial downcutting which extended the entrenchment of the South Nahanni River. Figure 3.03 on page 61 presents a map of the river.

The South Nahanni River meanders fairly regularly through four deep canyons within the southern MacKenzie Mountains. Small glaciers maintain the headwaters of the river. The fourth canyon terminates at its upstream end in Virginia Falls. The falls drop 300 feet (or 90 meters) to the canyon's base. The recession of the falls, during the past 300,000 years, is postulated to have been a significant factor in the development of the fourth canyon (Ford, 1972). Between canyons, the South Nahanni emerges as a braided river, but within canyon confinements it is very sinuous and meanders nicely.

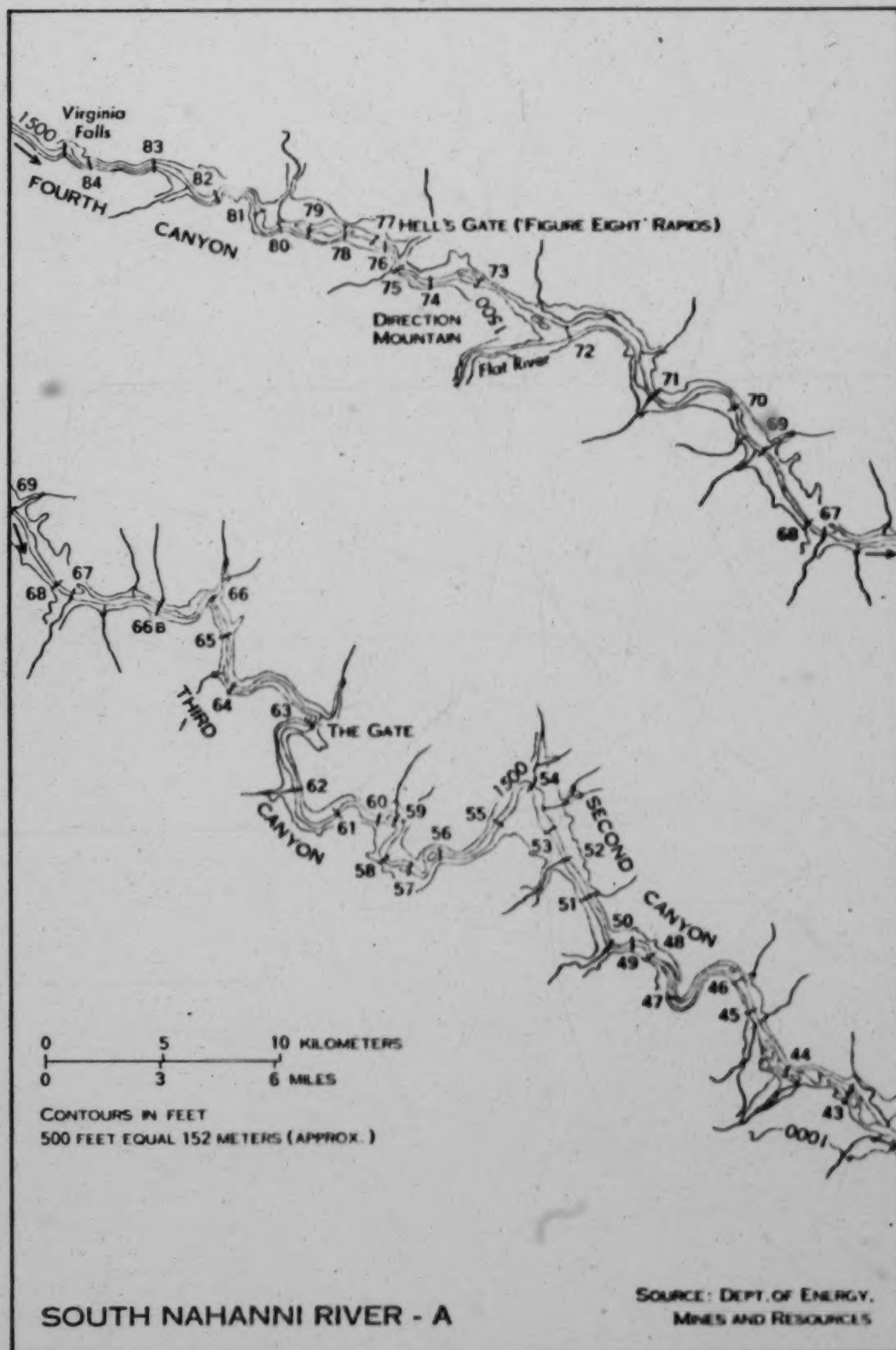


FIGURE 3.03A

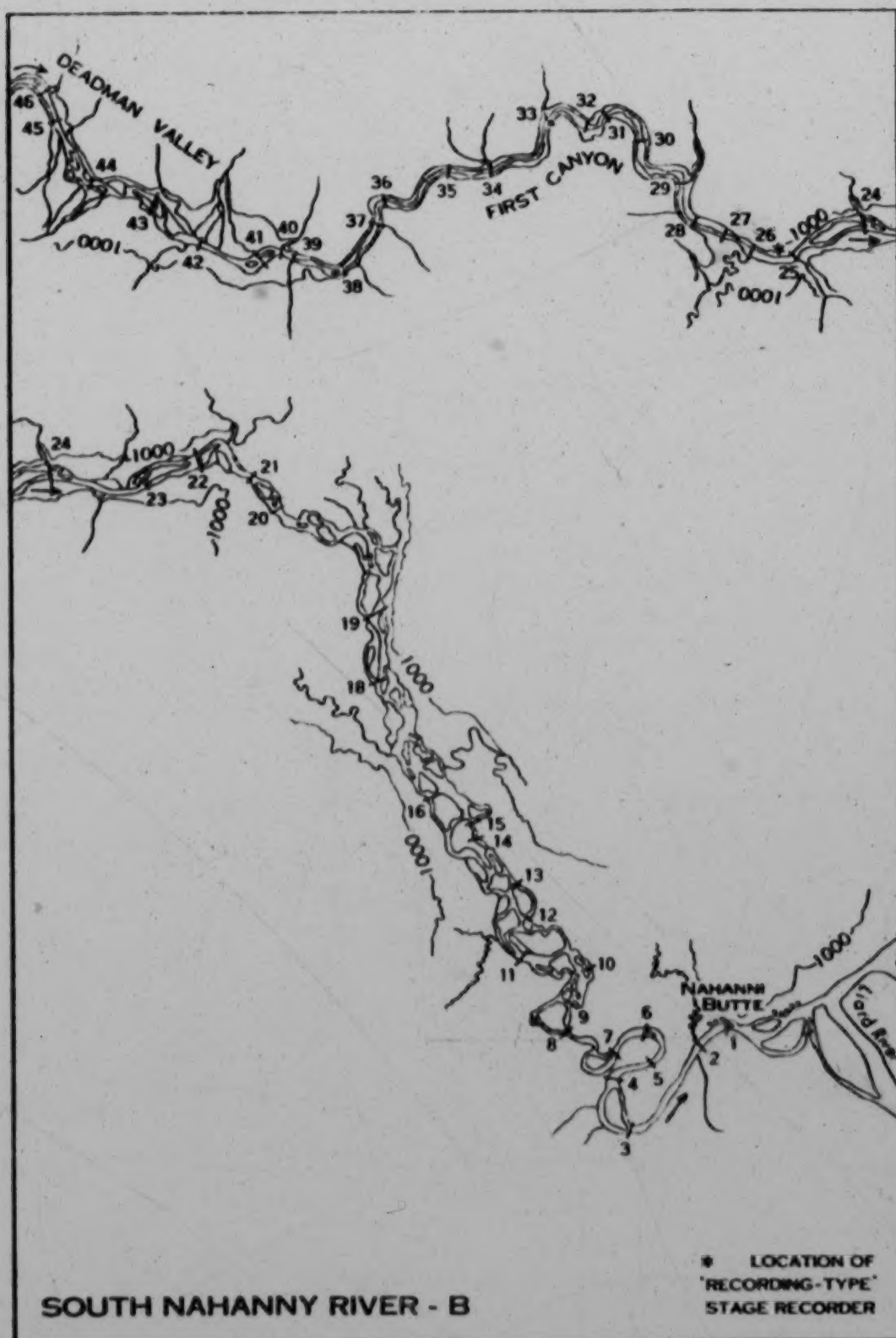


FIGURE 3.03B

Immediately downstream from the fourth canyon, within the first braided section but at a bedrock constriction, the flow path turns more than 90° --so sharply that the flow diverges, forming a set of "figure-eight" rapids, with large standing waves present at the bend. However, throughout most of the remainder of the South Nahanni's course, the flow is fast but sufficiently deep to prevent the formation of major rapids. Figure 3.14 reproduces a photo of the figure-eight rapids.

The depth of the canyon reaches varies between a few hundred feet and 3000 feet (or, between about one hundred and 900 meters). First Canyon is the local base level for the other canyons, and it is the deepest of the four canyons (Ford, 1970). Maximum relief within First Canyon is 3000 feet (or 900 meters).

The First Canyon is incised in a crystalline limestone formation (the Nahanni Formation) about 800 feet (or about 240 meters) thick, which is in places overlain by Simpson Formation Shales. The limestone is the main cliff-forming unit in the canyon (Ford, 1970). There are no obvious glacial features within the canyon. First Canyon meanders fairly regularly, as shown in Figure 3.10, with a sinuosity of approximately 1.5--the lower limit of "meandering" according to Leopold and Wolman (1960). Ford (1970) calculated the rate of canyon entrenchment, on the basis of dating of cave features and maximum relief, at no more than 2.66 feet (or, 0.8 meters) per thousand years, and he suggested that post-glacial uplift has likely accounted for at least 1600 feet (or 480 meters) of the canyon's relief. The gradient between Virginia Falls and First Canyon is very steep--about 2.25 per 1000.

b) Vegetation

The South Nahanni River region supports typical boreal forest stands. Atop its cliffs and on any islands that persist for sufficient lengths of time, the soil supports mainly spruce (Picea spp.), larch (Larix spp.), juniper (Juniperus spp.), wild rose (Rosa asicularis), and small plants such as bear-berry (Arctostaphylus uva-ursi), as well as a great variety of moss and lichens.

c) Climate

Table 3.01 lists monthly precipitation and temperature figures for the South Nahanni region. Maximum precipitation occurs between May and September, and temperature figures indicate that this adds to spring runoff in May and June. The river is normally frozen between October and early May. Climatic data suggest that the South Nahanni River maintains a high flow stage in early to mid-June, and so the survey of the South Nahanni was planned for mid-June.

d) Flow Information

The South Nahanni River flows south and east to join the Liard River which then follows a northeast path to join the north-flowing Mackenzie River, eventually emptying into the Beaufort Sea. The South Nahanni River drains an area of 12,900 square miles (or 33,000 Km.²).

A continuously recording gauge, operating since October, 1959, collects data on the flow of the river near Clausen Creek. Its location is marked in Figure 3.03 on page 61.

The mean annual discharge of the South Nahanni River is about 14,700 cfs. (or about 416 cu. m. per sec.) at the mouth of the First Canyon. (Surface Water Data, Water Survey of Canada) This average

discharge is very similar to that of the Colorado River in the Grand Canyon. Mean monthly discharge for June, the month of highest flow stage, is about 62,000 cfs. (or, about 1755 cu. m. per sec.). Extremes have occurred on June 17 or 18, 1962, when a maximum instantaneous discharge of roughly 290,000 cfs. (or 8210 cu. m. per sec.) flowed in the river, and on March 25, 1961, when only 1690 cfs. (or, 48 cu. m. per sec.) was recorded as the minimum daily discharge. Accurate sediment data is not available for the South Nahanni River, but flow is very turbid throughout the summer months, and a boulder bedload is observed to move through the canyons (Ford, 1970).

During the period of survey for this study (June 21 to 23, 1973) the mean discharge was approximately 24,798 cfs. (or 700 cu. m. per sec.), which is well above the mean annual figure.

3. The Kootenay River

a) Geology

The Kootenay River follows an irregularly meandering path semi-incised between the Stanford Range and the Rocky Mountain Trench. The map given in Figure 3.04 covers the reach of the Kootenay studied for this thesis.

Henderson (1954) published a detailed analysis of the geology of the Kootenay River region. For the convenience of the reader, the author's observations during the depth survey are supplemented with relevant information found in Henderson (1954).

The upper Kootenay River is entrenched in a broad, deep valley that follows a major fault zone southeast to a point just downstream from

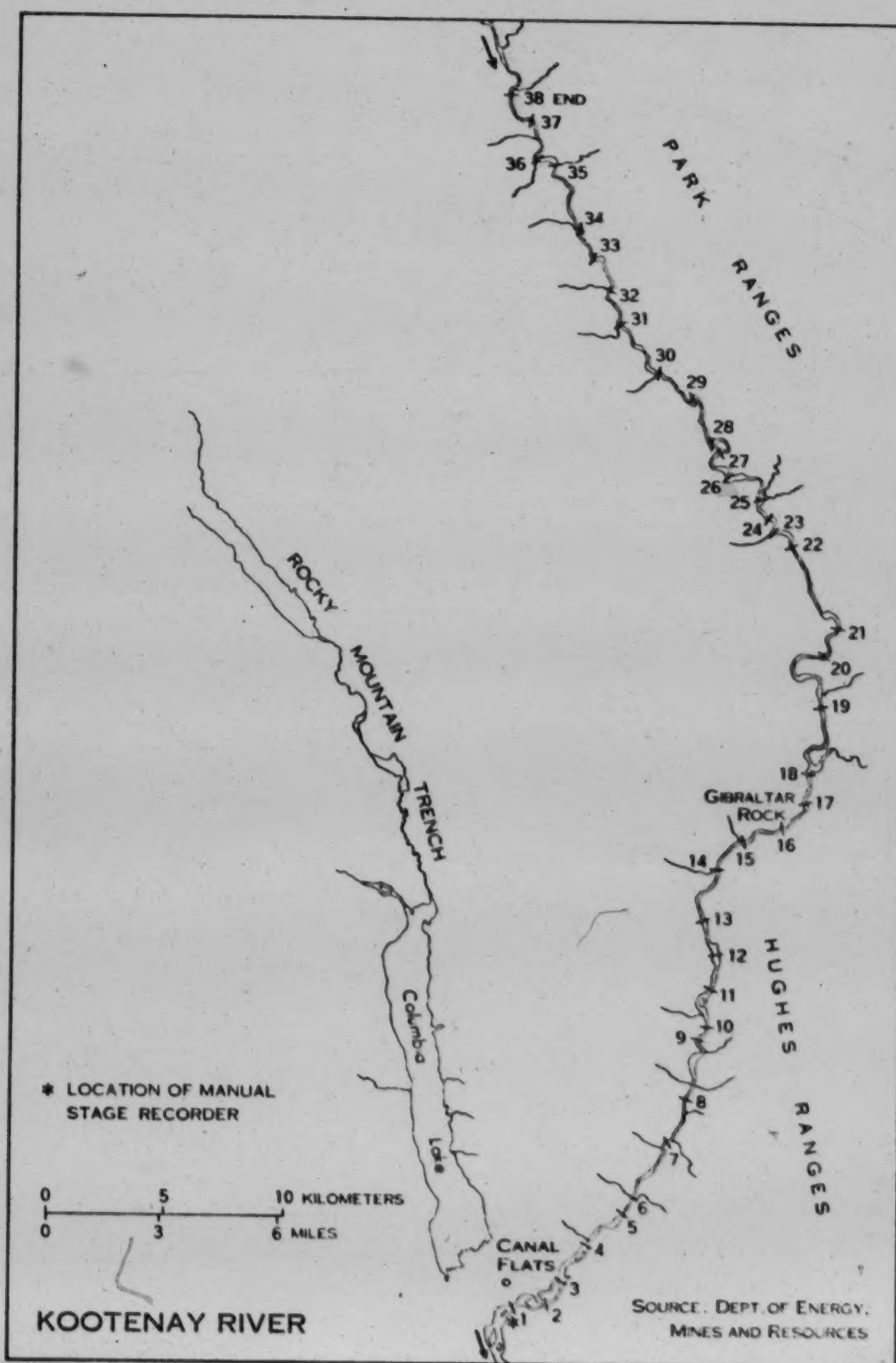


FIGURE 3.04

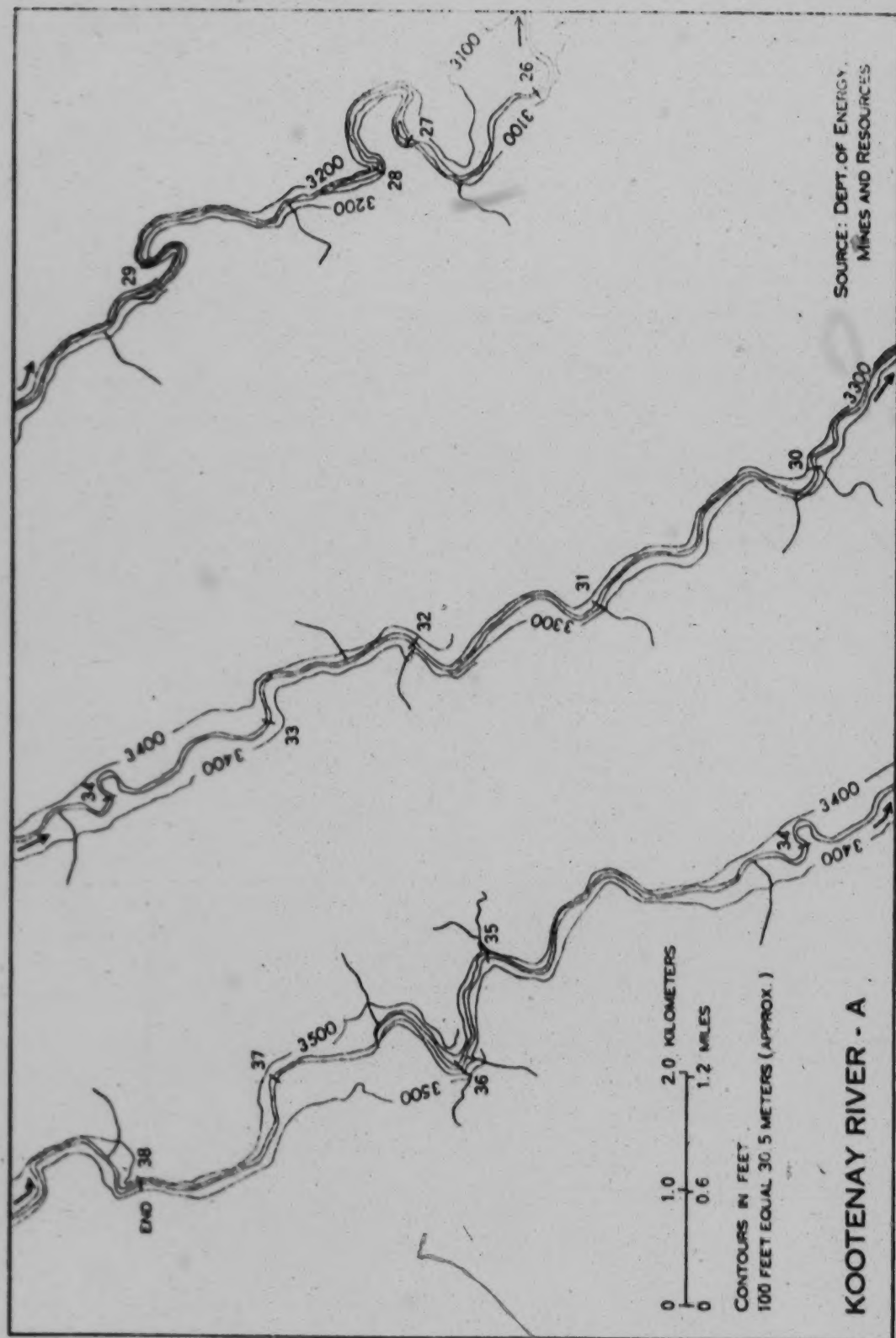
its confluence with the White River. The valley in which the upper Kootenay is incised maintains a fairly constant width of 2 miles at an elevation of 3250 feet. The river flows 50-300 feet below this main terrace level in its upper reach. Bedrock is visible in the banks. The upper Kootenay is, then, structurally controlled. Its general course is controlled by the occurrence of a narrow band of "soft calcareous chloritic phyllite" adjacent to the fault zone. (Henderson, 1954)

Below its confluence with the White River the Kootenay turns to flow southwest, crossing another structural trend obliquely. In this reach, the Kootenay forms a sharp canyon at Gibraltar Rock.

Further downstream, the Kootenay again changes its flow direction and proceeds south. The valley in this section becomes wider as the river passes over underlying gypsum. The new increased valley width is maintained until the Kootenay reaches the flats separating the southern extent of the Stanford range from the Hughes Range to the east. About 6 miles northeast of Canal Flats, the river tends slightly more towards the southwest and begins a braiding pattern which it continues much farther downstream. The river maintains its southwest flow direction adjacent an assumed oblique fault (Henderson, 1954) until it reaches the Rocky Mountain Trench.

Large outcroppings of gypsum (sedimentary rock gypsum--Henderson, 1954) occur along the Kootenay. Gypsum along the river reaches an exposed thickness of about 600 feet, and is interbedded with calcareous rocks of the Burnais formation (mainly limestone). Hoodoos occur along both banks of the river.

There is evidence of Pleistocene glaciation of the valley. Thick deposits of stratified silts and gravels occur along the valley, overlain



KOOTENAY RIVER - A

FIGURE 3.05A

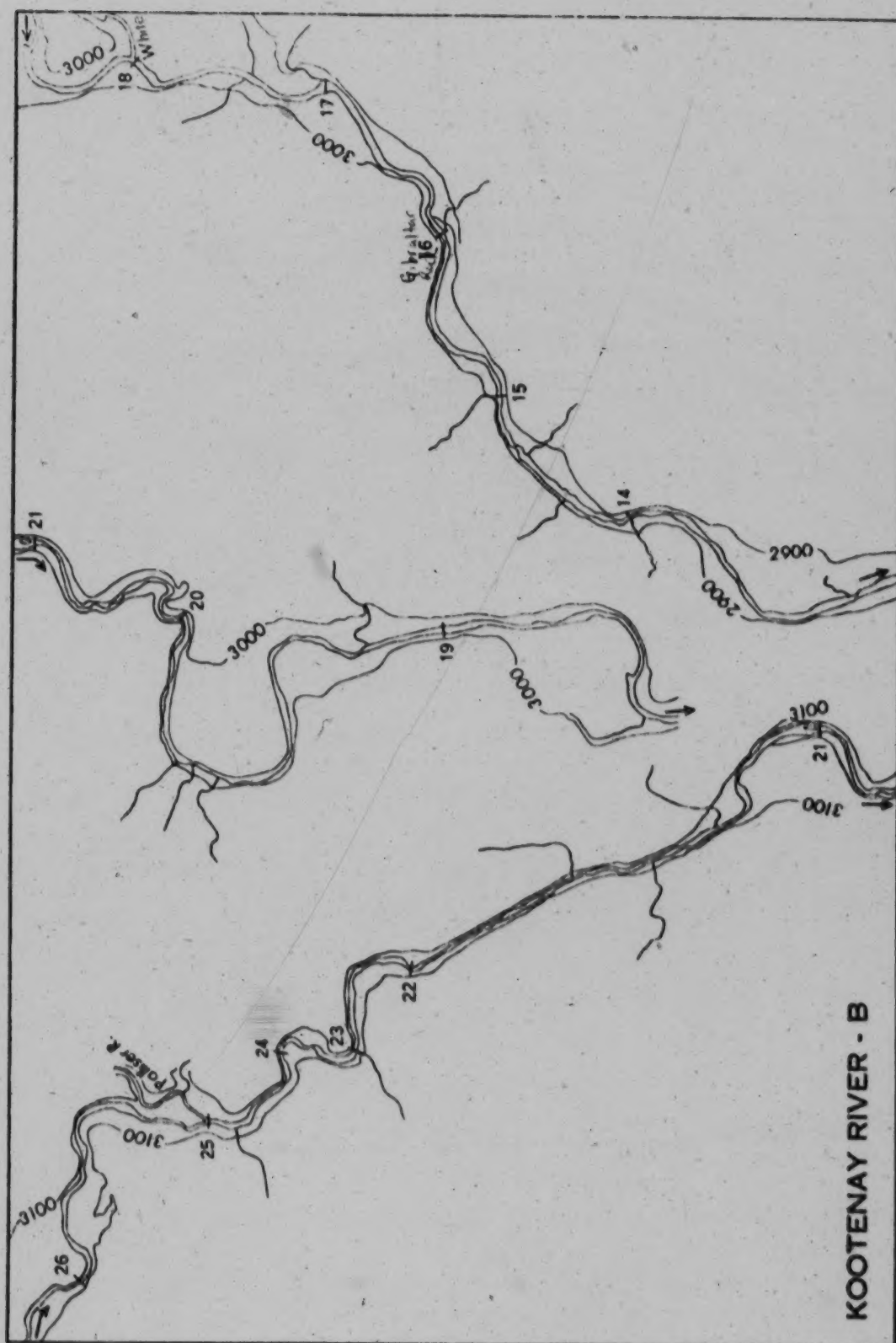


FIGURE 3.05B

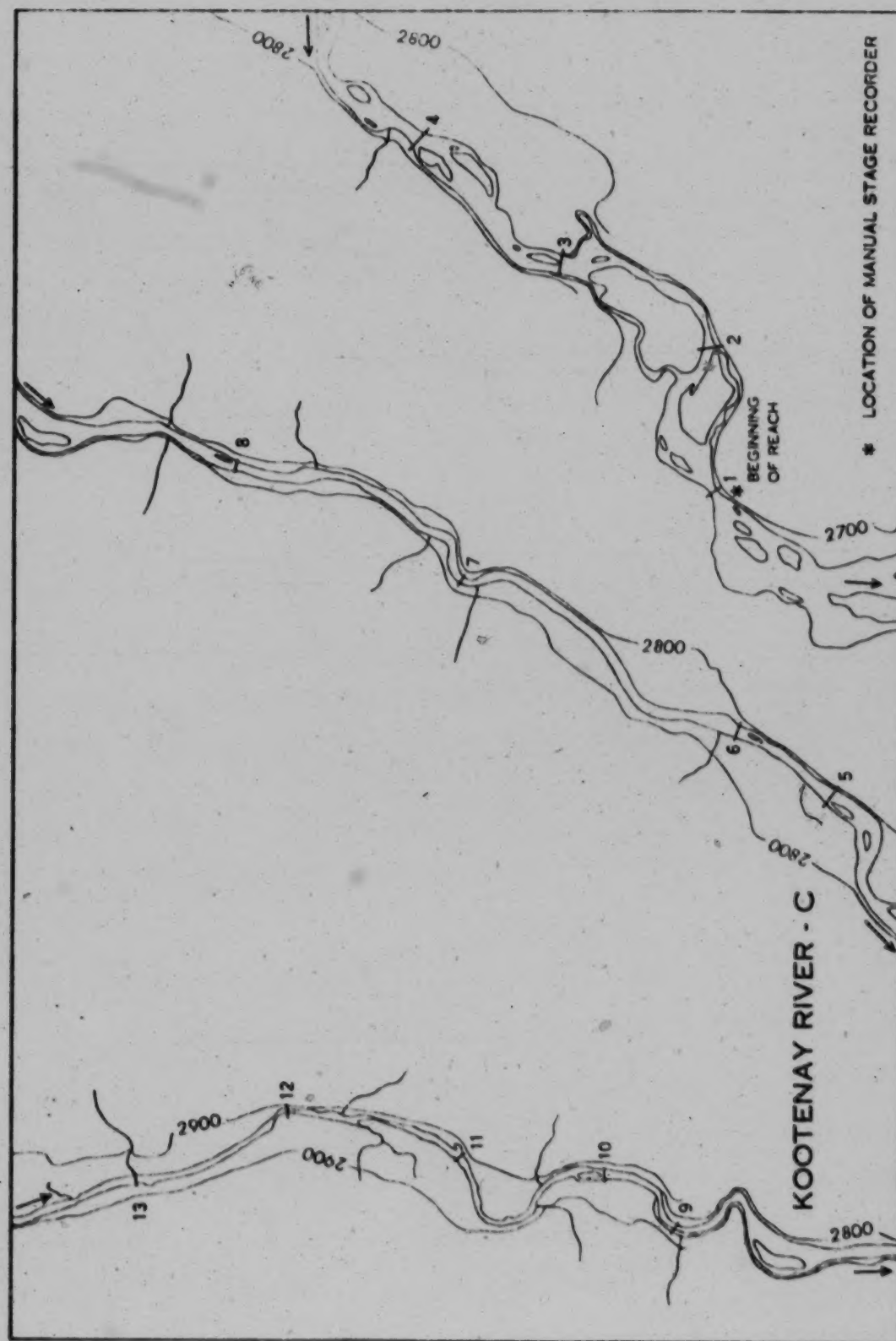


FIGURE 3.05C

by till in some places. However, the till is generally removed. Henderson (1954) suggests that the absence of u-shaped valleys, cirques, and hanging valleys in the Stanford Range seem to suggest that the area had been covered by a continental ice sheet that worked little to erode the rock.

b) Vegetation

Banks of the Kootenay are heavily forested, and extensive commercial lumbering is conducted in the region. Nearly pure stands of pine typify the regional forest.

c) Climate

Mean precipitation and temperature data for the Kootenay River area as listed in table 3.01, indicates that spring runoff normally begins sometime late in April and continues through May. Precipitation is relatively high during May, and high stage is thus expected to occur in late May or early June. Summer storms should cause increased flow for short periods, but late spring melt high in the mountain headwaters of the Kootenay River is the main peak stage regulator. The survey of the Kootenay was planned for the end of May and earliest part of June.

d) Flow Information

The Kootenay River flows south to join the Columbia River, eventually emptying into the Pacific Ocean. It drains an area of 2080 square miles (or 5370 Km.²). A manual gauge (read daily) near Canal Flats has been used intermittently from 1939 to 1943, from 1944 to 1950, and from 1963 to the present. Its location is marked in Figure 3.04

on page 66.

The mean annual discharge of the Kootenay River is about 3140 cfs. (or 90 cu. m. per sec.). Extremes have been recorded on May 24, 1948, when the discharge was 29,700 cfs. (or 840 cu. m. per sec.) and on April 1, 1941, when the discharge was a minimum of 178 cfs. (or 5 cu. m. per sec.). The mean annual flood occurs in late May or early June carrying about 19,000 cfs. (or 540 cu. m. per sec.). Bed materials for the reach studied were observed to be gravel.

During the time over which the survey was conducted for this study (May 31, 1973), the mean discharge was approximately 10,900 cfs. (or 300 cu. m. per sec.). This is well above the mean annual figure although it does not likely represent an annual flood.

G. DATA GATHERED - ϕ SERIES

1. Method of Measurement

In order to compare planform geometry with theories that suggest that sine generated curves, sine waves, circular arcs, or other geometric lines best model the meandering habit of rivers, angular deviations from the path of flow must be recorded at constant intervals along the path of flow (interval denoted Δx). A chord is thus constructed between each point, x_i , and its adjacent point, x_{i+1} , located x units downstream for the series of n points. In this way, $n-1$ chords are constructed for each river. The angles formed between adjacent chords are measured and these are listed in order, forming the series of deviation angles. Thus each deviation angle, ϕ , is a measure of the angle between the chord joining a point x_i and its adjacent downstream point, x_{i+1} , and the chord joining

the same point x_i and its adjacent point upstream, x_{i-1} . Figure 3.20, on page 95, explains the measurement of deviation angles in greater detail. Each ϕ_i , then, depends on the location of the preceding point for its measurement, but is independent of any fixed origin or baseline for the series.

Some interesting and apparently previously unrealized consequences of this manner of obtaining meander data will be discussed in Chapter 5, but it can be pointed out here that this is the method used by Speight (1965), Chang and Toebes (1970), Surkan and Van Kan (1969), Thakur and Scheidegger (1970), and Church (1972). These papers constitute all work published to date applying spectral analysis to the planimetric geometry of meandering rivers.

This method of measurement for deviation angle of flow is an ideal parameter of meander planform for many reasons. The parameter ϕ_r (angular deviation in radians) can be directly related to the radius of curvature R (derived as in Figure 3.20), for any segment of the river, as:

$$R = \frac{1}{2} \frac{\Delta x}{\sin(\frac{\phi_r}{2})}$$

This R can be used directly in the calculation of the centrifugal acceleration, a , of the flow as:

$$a = \frac{V^2}{R},$$

where V is the flow velocity. Flow velocities were not recorded in this study, and so this concept has not been brought to use in this thesis.

The use of flow path rather than specific origins or base lines (ie: down-valley Cartesian co-ordinates) in the calculation of

ϕ_i eliminates the old problem of the possibility of multi-valued (non-functional) relationships. At the same time, it overcomes problems often encountered in the determination of a single flow direction or valley orientation to describe a river's path. A general valley orientation or baseline often becomes meaningless with respect to local flow direction, as the river's flow path can become complicated for tortuous or even very sinuous reaches.

An arbitrary convention for this work designates left turns relative to the downstream direction along a river as negative curves, and right turns are considered positive curves. For convenience in later analyses, when removal of the mean is necessary for further calculations, right turn values ($+\phi_i$) may be added to an arbitrary mean value, and left turn values ($-\phi_i$) may be subtracted from a convenient mean, establishing a new non-negative series of ϕ_i values. In this study, in order to facilitate computer coding, means chosen were 100 for the Beaver and South Nahanni Rivers, and 200 for the Kootenay River. Thus in illustration, for the Beaver, a value of 102 indicates a 2° turn to the right in a downstream direction, while a value of 95 indicates a 5° turn to the left, as measured between adjacent chords.

2. Thalweg Problems

The river thalweg is the ideal line on which to base the measurement of both stream depths and planform angles. The thalweg traces the river's own main channel (and only channel at extremely low discharges) and so characterizes the sculpturing ability of rivers with respect to their plan and depth. However, information on the precise location of the thalweg is not available for many rivers. Few maps contain such

data, and low air photographs provide accurate information or detailing to expose the location of the thalweg. Fortunately, the midstream flow path adequately approximates the thalweg line, within the limits of accuracy allowed by most map scales, and so can be substituted for thalweg in the calculation of ϕ_1 . That is, the difference between the locations of the thalweg and midstream paths introduces little error at the scale of measurement possible when common maps or air photos are used.

The problem of thalweg location is less difficult in the field, and so is more easily solved for the depth series. However, for large rivers, safe navigation of the thalweg during depth recording may often not be possible within the limits set either by the required accuracy of depth data or the navigator of the boat. Thalweg depth traces for this study were only gathered for the Beaver River.

3. Maps and Air Photographs

Adequately scaled maps and air photographs were available for the three rivers studied. All maps used were NTS maps, and all maps and air photographs were made available by the Federal Department of Energy, Mines, and Resources.

The survey of the Beaver River, involving an 18.5 path-mile reach from the Alberta-Saskatchewan border to the Beaver Crossing Bridge, required only one map at the 1:50,000 scale. NTS map 73 L/8, compiled in 1968 from air photos taken in 1960, was found to be sufficiently accurate for the compilation of both the meander and the depth series for this study. Figure 3.02 gives the over-all view of the reach, including detailed plans and position fixes.

Maps of the South Nahanni were not available at the 1:50,000 scale. The best maps available were the NTS quadrangles of Virginia Falls (95F) and Sibbeston Lake (95G) at the scale of 1:250,000. These provide only crude location references. However, air photos of the entire reach were available at 1:1320 and these were adequate for locational information. Photos, taken in 1956, are listed in table 3.03. The survey included 123.1 miles of the South Nahanni River. Figure 3.03 on page 61 gives the overall view of the reach, including detailed plans and position fixes.

Maps were adequate for the survey of the Kootenay River. General location was provided by NTS 82J at 1:250,000, while position fixes along the river were made using the 1:50,000 NTS series including 82 J/12 W, 82 J/12E, 82 J/5E, 82 J/4E, and 82J/4W. The survey included 46.1 miles of river from Canal Flats bridge to the "cable car crossing" upstream (approximately $50^{\circ} 45' 07''N$, $115^{\circ} 45' 04''W$). Figure 3.04 gives the general view of the reach, while Figure 3.05 gives detailed plan and position fixes on the river.

In all cases, for the planform series, x was chosen as one-tenth mile. The interval seemed to adequately cover river meanders while it simplified calculations. In retrospect, it probably would have been advisable to choose x as a function of river width, and this will be done in future research. The midstream line was chosen as the best estimate of river path, and deviation angles were thus found at intervals of 0.1 miles along each river's centerline.

River	Map Specifications	Map Scale	Air Photos
Beaver River	NTS 73 L/8, Edition 3 MCE, series A 741	1: 50,000	
South Nahanni River	NTS 95 F, Edition 3 MCE, series A 502 and NTS 95G, Edition 2 ASE, series A 502	1: 250,000	A17620: 82, 84, 86, 88, 90, 92, 94 A17434: 158, 156, 154, 152, 150, 148 A17620: 114 A17434: 139, 137, 135, 133, 127, 125, 123, 114, 116
Kootenay River	82J, Edition 1 ASE, series A 502 82J/12W, Edition 1 ASE, series A 721 82J/12E, Edition 1 ASE, series A 721 82J/5E, Edition 1 ASE, series A 721 82J/4E, Edition 1 ASE, series A 721 82J/4W, Edition 1 ASE, series A 721	1: 250,000	

Table 3.03 Maps and Air Photos Used for the Study

D. DATA GATHERED: DEPTH SERIES

Continuous records of each river's longitudinal profile were obtained for the purposes of this study. Continuous depth echo-sounding recorders were mounted in the boats, which then proceeded along the river on a path that maximized potential utility of output in relation to problems associated with each survey.

A sixteen foot Larsen fiberglass boat, supplied and operated by the Research Council of Alberta, was used to survey the Beaver River. The boat was equipped with a side-mounted transducer which sounded depths with a beam angle of 15° . The recorder used in the survey was a DE 719 Raytheon, and a chart speed of two inches per minute was maintained throughout the survey. The original depth chart for the 18.5 mile survey (series D) was 29.2 feet in length.

Although a completely vertical sounding would have been ideal for the survey, it was felt that the error incurred using the slightly foreward-angled transducer was insignificant for the purposes of this study. A thalweg depth trace was obtained for the Beaver River. The survey was conducted on June 22, 1973, approximately between 1500h and 1900 h Mountain Daylight Time.

Twenty-eight position fixes were recorded along the upstream path followed, and these were marked on the topographic maps and depth charts, but it was later found that two fixes were not accurate. Both fix problems were attributed to human error in the identification of landmarks, and when omitted, did not seriously affect the record obtained.

Figure 3.06 on page 79 shows, for a meandering section of the Beaver, the actual photographs used in this study, the depth soundings

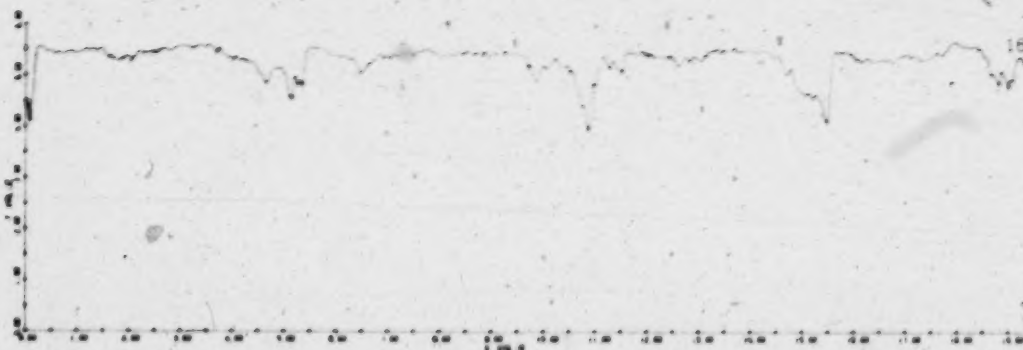
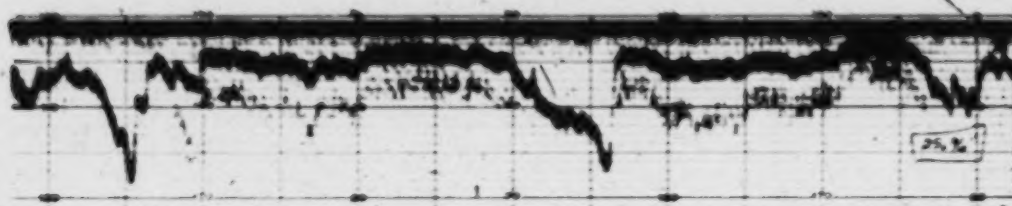
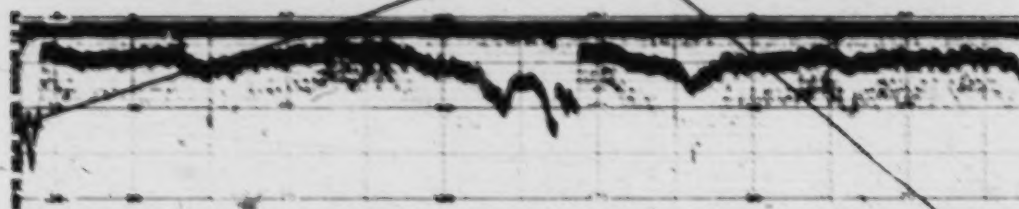


Figure 3.06 Beaver River: section of typical regular meanders. Reach marked on air photo (top) extends upstream from mile 5.4 to mile 6.7. Chart (middle) shows longitudinal depth profile recorded as boat moved upstream. Cal camp plots (bottom) give digitized depth series adjusted to run's standard boat speed.

obtained, the position fixes used, and the velocity-corrected record for the same blocks. The correction procedure is outlined later. Figure 3.07 on page 81 illustrates the same qualities for a straight section of the river.

Reconnaissance work on the Kootenay River indicated the necessity of a larger boat, and so a twenty-six foot Dorian jet boat, again supplied and operated by the Research Council of Alberta (constructed by Northwest Jetdrive in Edmonton), driven by a 455 h.p. Oldsmobile engine, was used for the Kootenay survey. The jet boat was equipped with both a Raytheon DE 719 continuous depth recorder attached to a hull mounted transducer (which again sounded depths with a beam angle of 15°) and a digital output second depth meter, which was useful in indicating the thalweg to the pilot. The Kootenay River survey was conducted on May 31, 1973, approximately between 1000 h and 1600 h Mountain Daylight Time (see Figure 3.19).

The chart speed was again set at two inches per minute, and about forty feet of chart were obtained in the upstream run (Series C). Thirty-seven position fixes were marked, and all were later used.

Unfortunately, the Kootenay River was found to be too rough to permit a thalweg survey while maintaining safe operating conditions. A record of midstream depths was obtained instead (see Figure 3.18).

Figure 3.08 on page 82 shows the air photograph of a curving section of the Kootenay. The original depth record for the blocks indicated and the boat-velocity corrected depth record are also shown. For contrast, Figure 3.09 on page 83 shows a fairly straight, narrow incised rock section of the Kootenay. Original and corrected depth records are included for the straight section, as indicated.

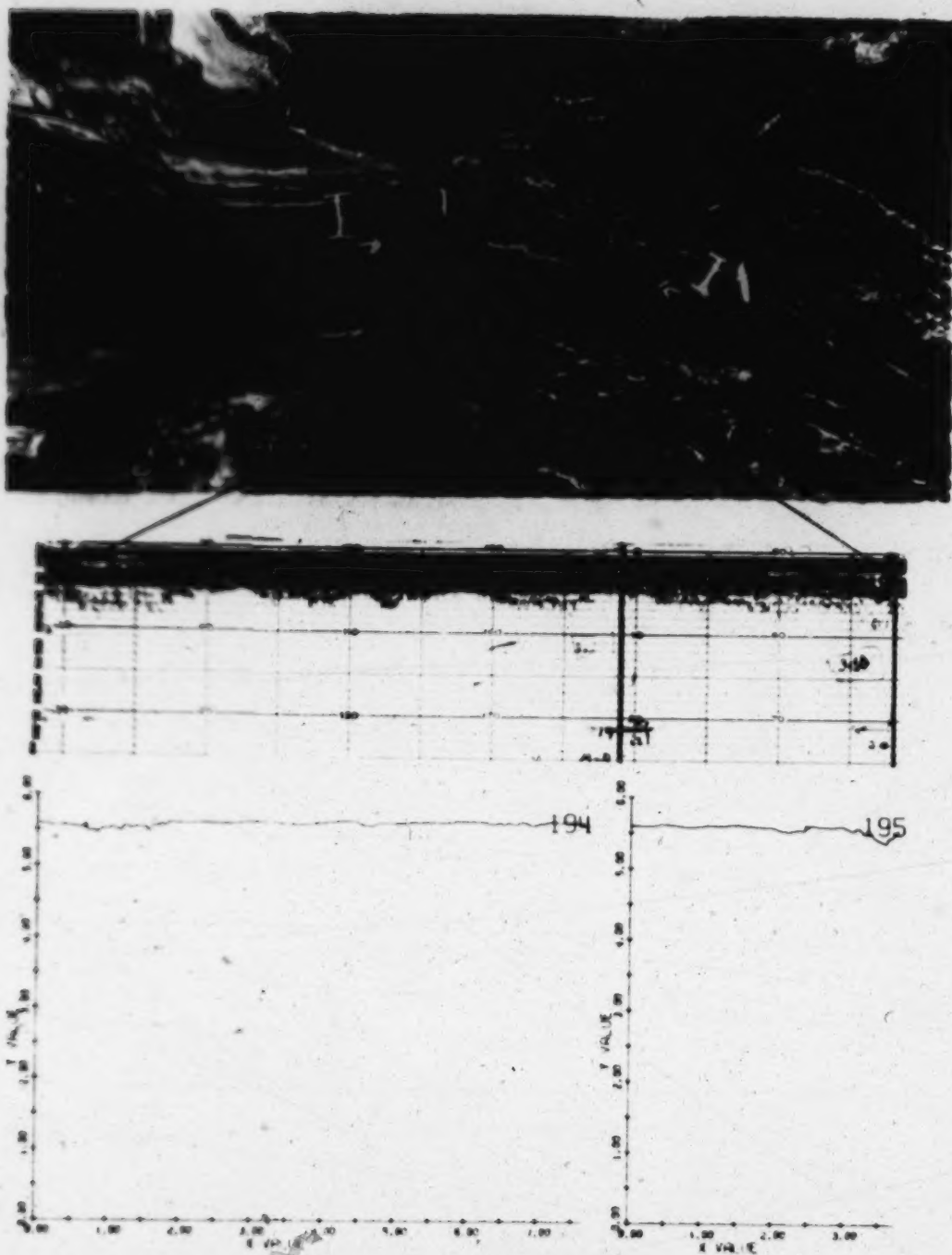


Figure 3.07 Beaver River: straight section. Reach marked on air photo (top) extends upstream from mile 11.7 to mile 12.4. Chart shows longitudinal depth profile recorded as boat moved upstream (series D). Calcomp plots (bottom) give digitized depth series adjusted to run's standard boat speed.

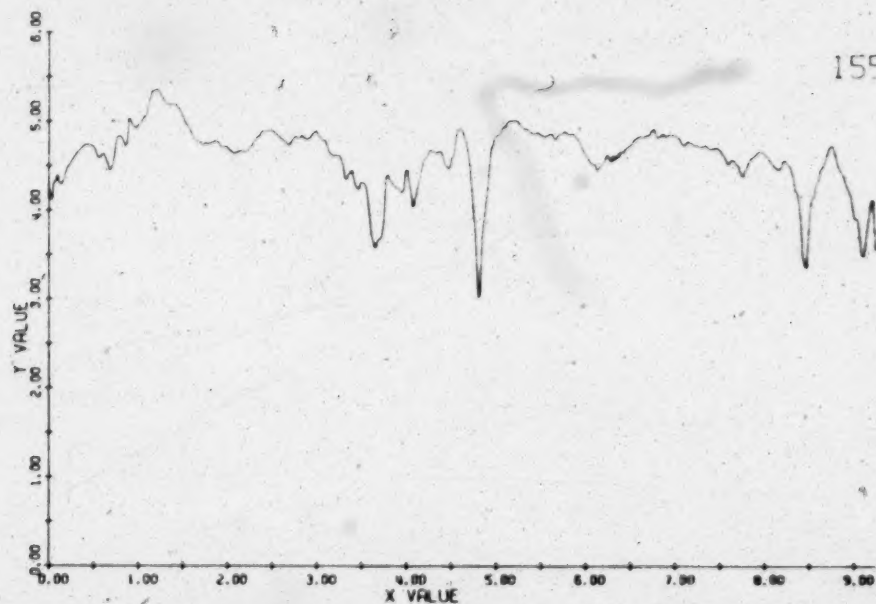
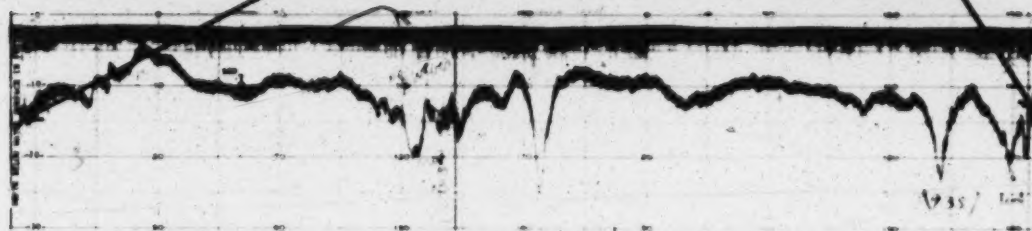


Figure 3.08 Kootenay River: entrenched section with some meandering. Reach marked on air photo (top) extends upstream from mile 16.2 to mi. 16.4. Chart (middle) shows longitudinal depth profile as boat moved upstream (series C). Cal comp plots give digitized depth series for reach, as adjusted to standard boat speed for run.

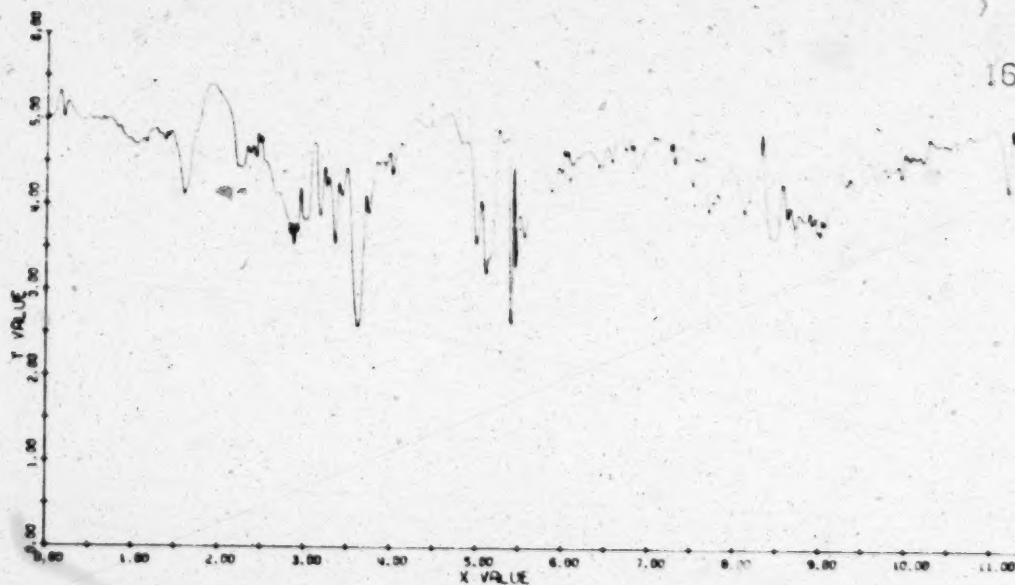
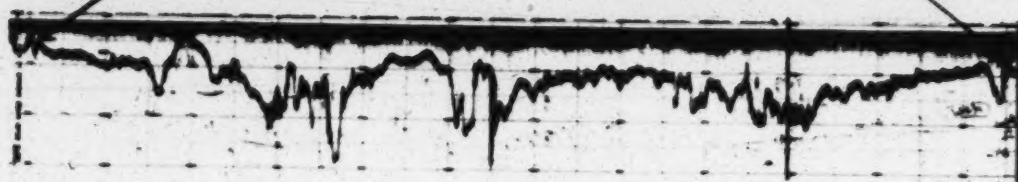


Figure 3.09 Kootenay River: entrenched section with straight rock canyon. Reach marked on air photo (top) extends upstream from mile 26.5 to 29.2. Chart (middle) shows longitudinal depth profile, recorded as boat moved upstream. Cal comp plots (bottom) give depths for reach, adjusted to run's standard boat speed.

Records were obtained for the South Nahanni River both as the boat proceeded upstream and downstream (series A and B, respectively). For the Nahanni surveys, a twenty-six foot jetboat was rented from the South Nahanni River Expeditions Company, based in Fort Simpson, N.W.T. A tube type Raytheon depth recorder was used, supplied by Water Survey of Canada, with a side-mounted transducer, again having a beam width of 15° . An upstream survey of the 127 mile reach began June 21, 1973, at approximately 1600 h, continuing until about 2200 h the same day. On the second day, the survey began about 1000 h and continued in heavy rain all day. A downstream survey was conducted on June 23, beginning about 1000 h and continuing until about 1900 h. Chart speed for both upstream and downstream surveys was set at 36/60 inches per minute. The downstream chart produced was only 4 feet long, and over 13 feet of chart were recorded during the upstream run (see Figures 3.16 and 3.17).

Position fixes were taken for both the upstream and downstream runs, in identical places where possible, and a total of eighty-four were recorded during the longer upstream survey. A number of the fixes recorded in the braided sections were later found to be unusable. Island changes during the time since the original production of the air photographs confused the two navigators. Landmarks were not identifiable from photos, and there was no immediate solution to the problem. Consequently, individual data "blocks" are very long for the braided section.

A thalweg survey was considered unfeasible for the South Nahanni River. The boat operator's original opinion was that the thalweg would be too swift to allow safe collection of data. In retrospect, however, it seems that a thalweg trace could probably have been obtained for the

river, and it is unfortunate that this was not done. Instead, the river's "anti-thalweg" was followed in order that quieter water might be navigated. Figure 3.15 exemplifies the rough-water problem.

Figure 3.10 on page 86 shows, for the meandering section of the First Canyon, the blocks used, depth chart obtained, and corrected depths. Figure 3.11 on page 87 provides similar information for the Second-Third Canyon areas. For the braided sections, Figure 3.12, on page 88 shows these features. (In this figure, block #10 appears elongated on the chart, as a slower chart speed was attempted for this block only.) "The Gate", a spectacular narrow canyon in Third Canyon is shown in Figure 3.13, on page 89. Figure 3.13 again gives both the original and the velocity-corrected depth record for this section of the downstream run. It was not possible to obtain a record for this reach in the upstream run as velocity problems overcame the depth recorder.

The Digicon digitizer, built by the University of Alberta Technical Services Department, was used to digitize and file the depth chart data. In the digitizing process, the chart is placed on a magnetic digitizing table. A programmer then traces over the actual record with a moving arm that magnetically records its position to the nearest specified interval. Charts in this study were digitized at a Δx of one-fiftieth of an inch. It was felt that no usable information was present in smaller wavelengths than this sampling interval provided. Individual blocks (the chart lengths between subsequent fixes) were digitized separately and stored in separate files on a single magnetic tape. The digitized record was transferred to an IBM compatible 9 track magnetic tape for permanent storage.

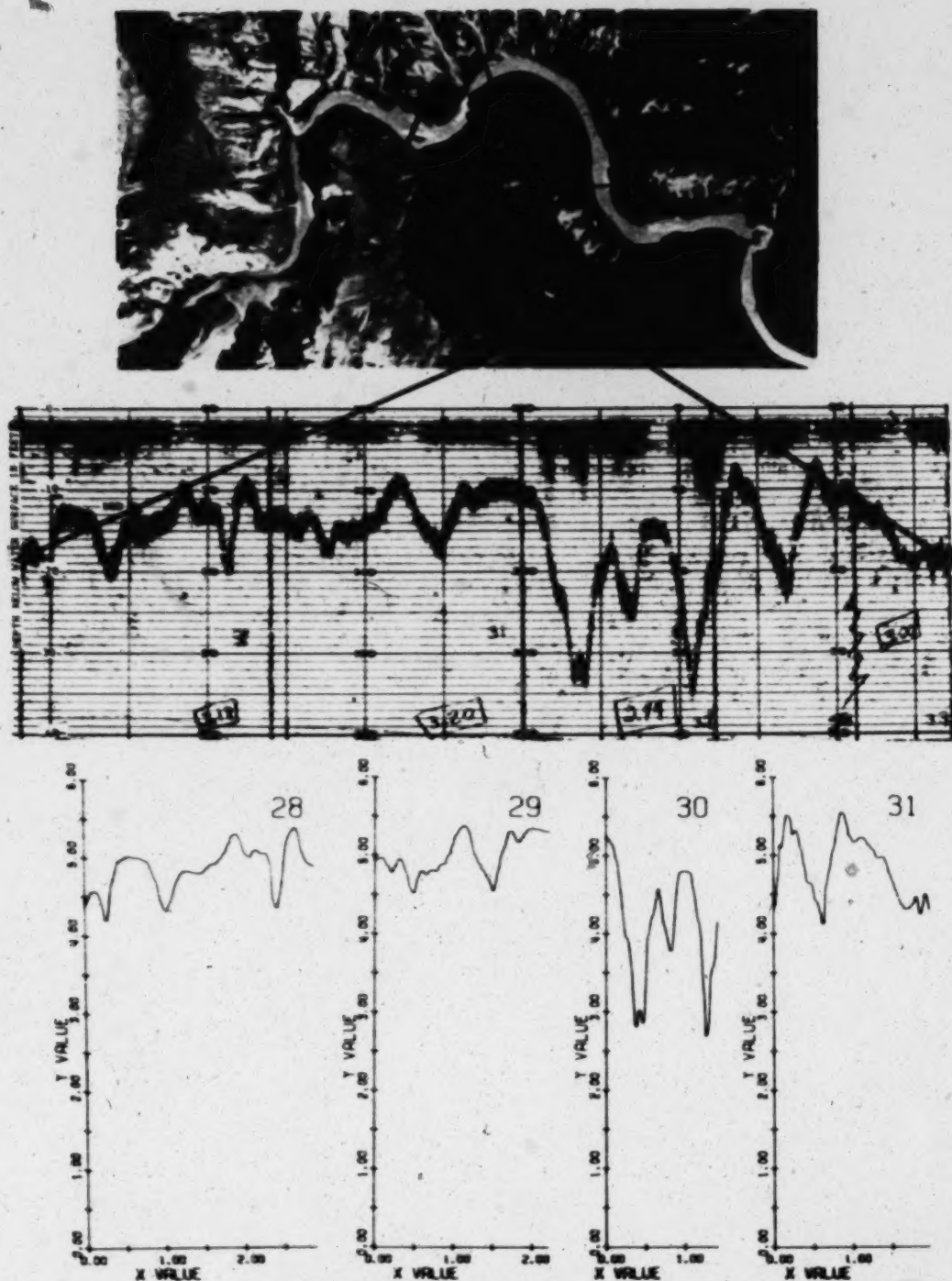


Figure 3.10 "First Canyon" of the South Nahanni River. Reach marked on air photo (top) extends upstream from mile 82.2 to mile 76.9 below Virginia Falls. Chart (middle) shows longitudinal depth profile for reach, recorded as boat moved upstream (series A). Cal comp plots at bottom give digitized depth series adjusted to standard boat speed.

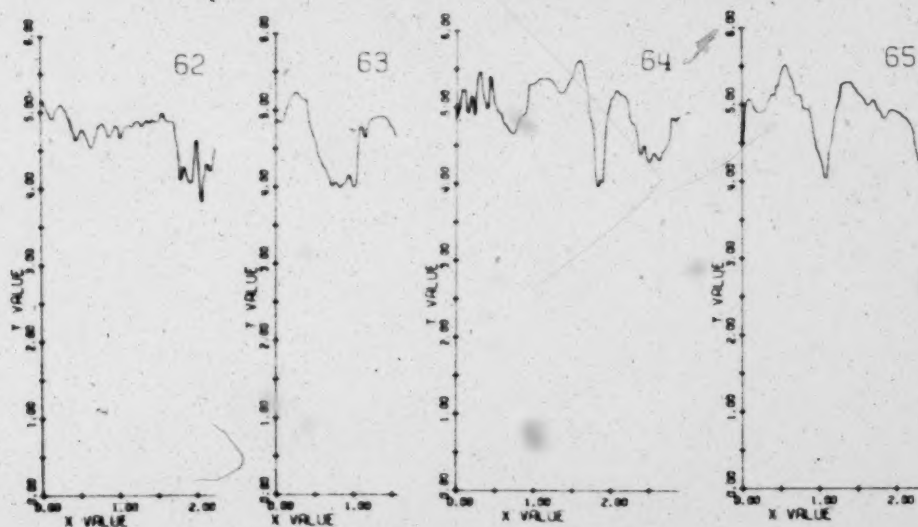
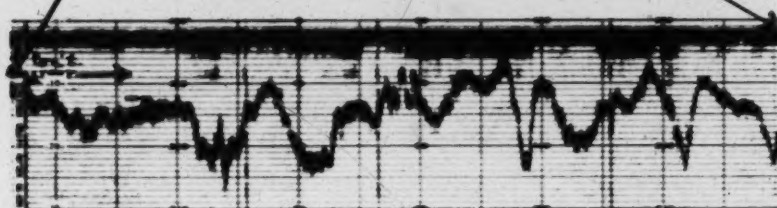


Figure 3.11 Second and Third Canyons on the South Nahanni River. Reach marked on air photo (top) extends upstream from mile 33.3 to 27.4 below Virginia Falls. Chart (middle) shows longitudinal depth profile recorded as boat advanced upstream (series A). Cal comp plots (bottom) give digitized depth series adjusted to standard boat speed for series.

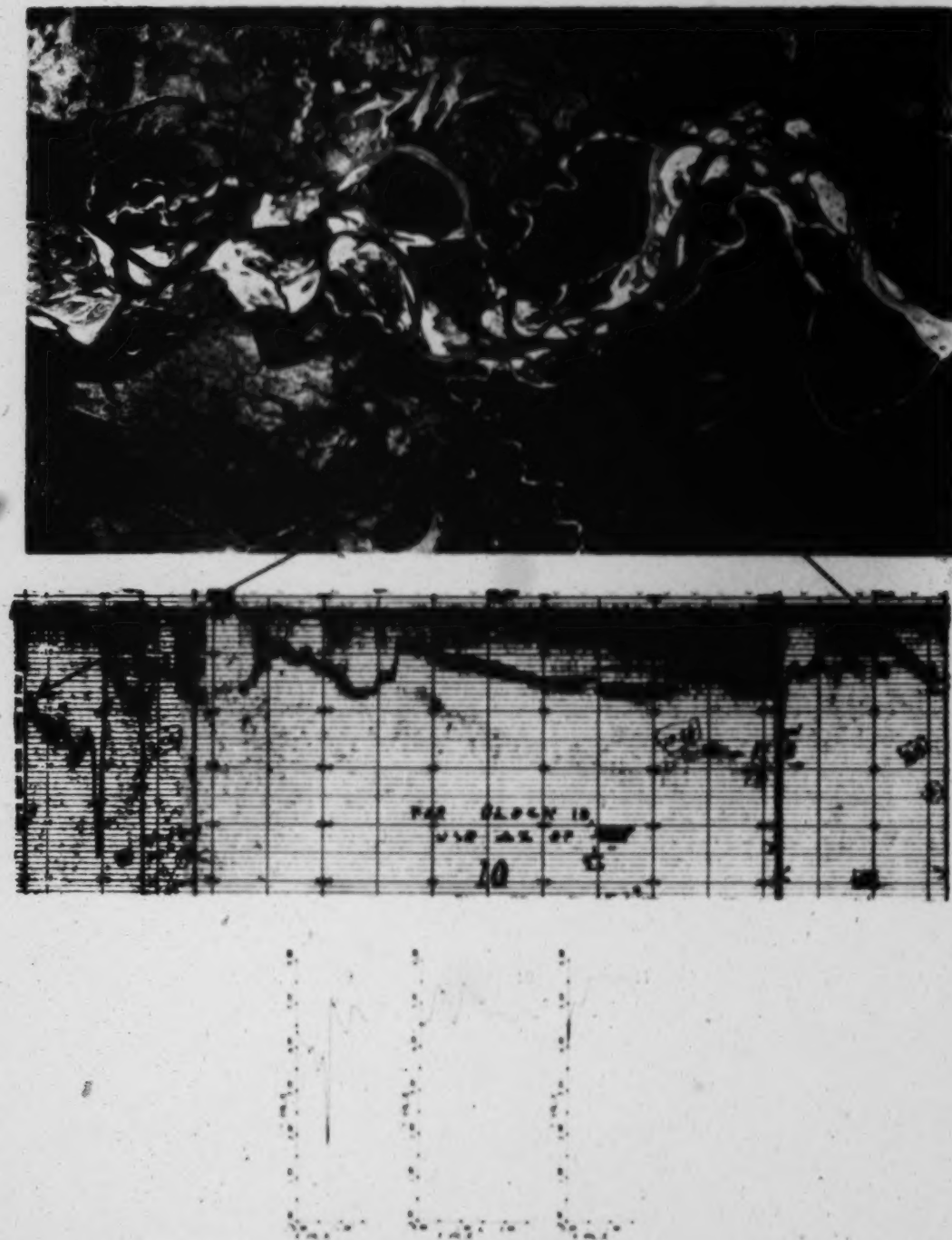


Figure 3.12 South Nahanni River braided section, near Liard confluence. Reach marked on air photo (top) extends upstream from mi. 114.2 to 108.8 below Virginia Falls. Chart (middle) shows longitudinal depth profile recorded as boat advanced upstream (series A). Cal comp plots (bottom) give digitized depth series adjusted to standard boat speed for run A.

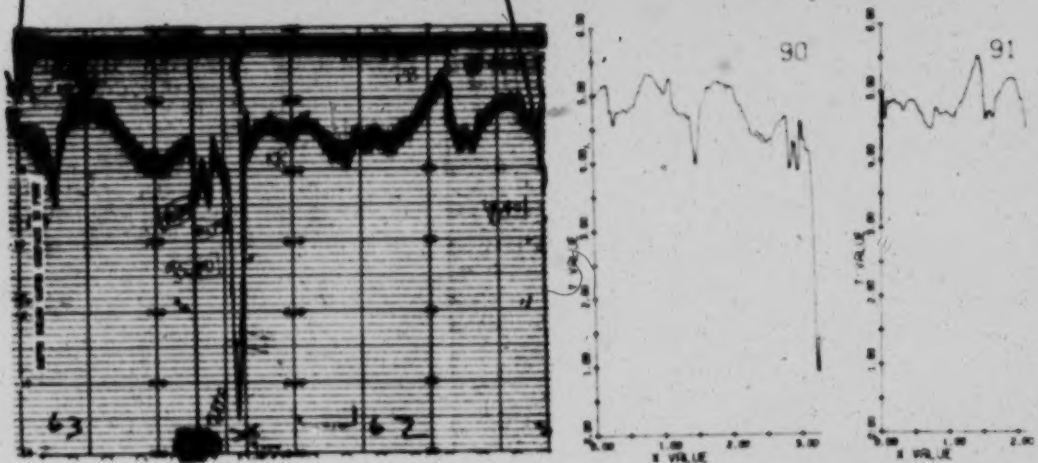
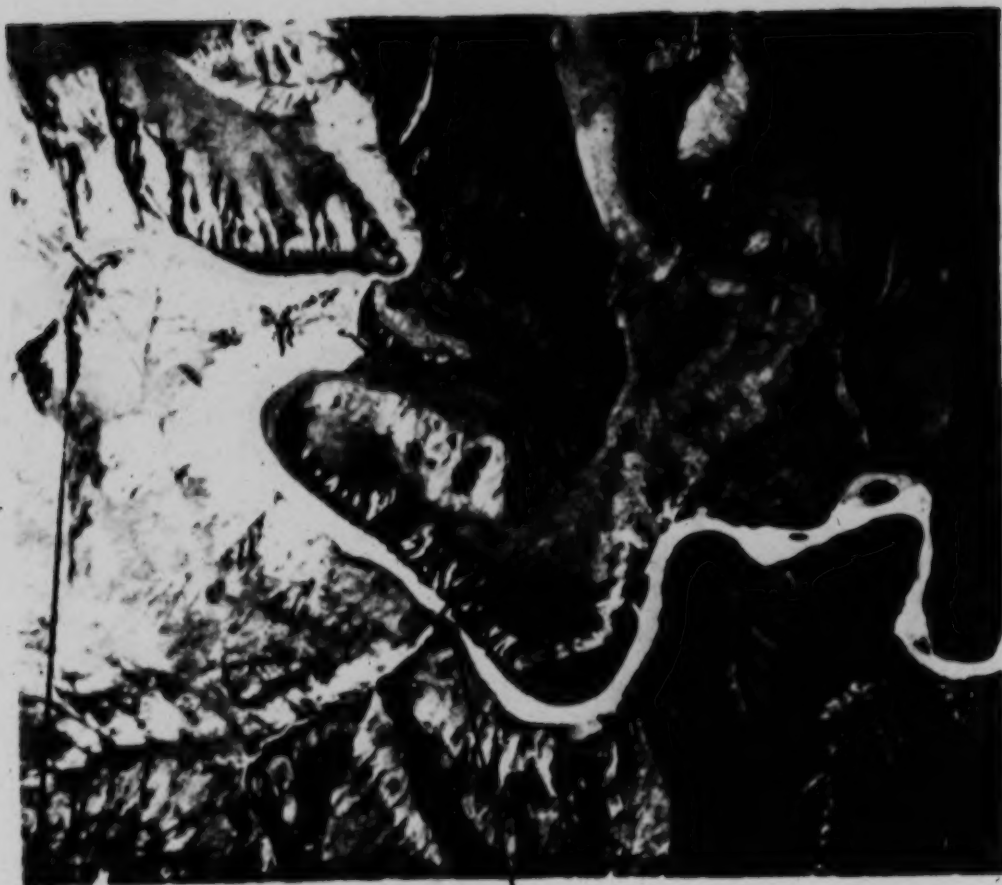


Figure 3.13 "The Gate" in Third Canyon on South Nahanni River. Reach marked on air photo (top) extends downstream from mile 33.2 to 40.6 below Virginia Falls. Chart (left) shows longitudinal depth profile recorded as boat moved downstream (series B). Cal comp plots (right) give digitized depth series adjusted to standard for river.

E. DATA GATHERED: CORRECTED DEPTH RECORD

It was necessary, in the initial stages of analysis, to keep individual blocks separate, as it was known that the boat speed must vary along the path travelled (both for logistic and safety reasons). As the boat proceeded upstream, shallow areas would force the boat to proceed with a forward velocity slow relative to its mean velocity. Deep pools in the path would cause the boat to move relatively quickly. The depths were recorded at a constant time interval by the depth recorder, and thus at higher boat speeds, the same length of river would produce less chart than would be obtained at slower speeds. It was necessary to correct for this error during the data analysis. Since the ratio of the river distance-travelled to the corresponding chart distance obtained could be found for each block (between two known fixes) the mean velocity travelled along any block's river distance could be easily computed as $\frac{\text{distance travelled}}{\text{time elapsed}}$. Elapsed time was found in terms of inches of chart comprising any block compared to the corresponding chart speed.

Once the mean boat speeds were calculated for each block, the block corresponding to the fastest mean velocity in any survey could be determined. All other blocks in the survey then contained relatively too many points, compared to the "standard" block, and points could then be systematically removed from all other blocks to correct for velocity fluctuations.

The number of points removed was proportional to the ratio between points per unit distance along the river in the standard (fastest) block and that of the block compared. It was felt that a systematic removal of points would not significantly modify the areas of interest

of the spectrum of the block involved, considering that the x of the chart record digitizing was one-fiftieth of an inch. Point removal conducted systematically only corrects the record, so that blocks can finally be joined as a continuous series. Appendix 1 on page 185 lists the original number of data points in each block and the corresponding percentage of these removed by this process. The removal process is outlined in further detail in the appendix. Figures 3.06 - 3.13 allow visual comparison of many blocks before and after point removal. The figures show well the success of the method outlined, and the lack of "loss" of relevant data. An alternative method of correcting for boat speed fluctuation would be to "stretch" some (or even all but the slowest block) blocks using some means of interpolation. Either (or both) method would probably give satisfactory results, but point removal is simpler and no significant information is lost if x is sufficiently small.

Once the corrected continuous data series was obtained, tape files were corrected, and the final correct series were stored. This data may now be obtained upon request, from Ms. D. Dodd, Programme Analyst, Geography Department, University of Alberta. A blank magnetic tape should be supplied with each request. Data will then be duplicated onto the tape, and the tape will be returned as directed. Alternatively, the uncorrected data may also be obtained. The original chart records will be given to and stored by government agencies (Beaver: Research Council of Alberta: Edmonton; Kootenay and South Nahanni Rivers: Water Survey of Canada.)



Figure 3.14 "Figure-eight" Rapids on the South Nahanni R.



Figure 3.15 Rapids on the South Nahanni R. -- series A, B



Figure 3.16 Transducer mount for South Nahanni R.-- using S. Nahanni River Expeditions Ltd. jetboat for surveys A, B



Figure 3.17 Transducer in operation; guy line to front of boat is underwater



Figure 3.18 Standing waves on the Kootenay R. -- series C

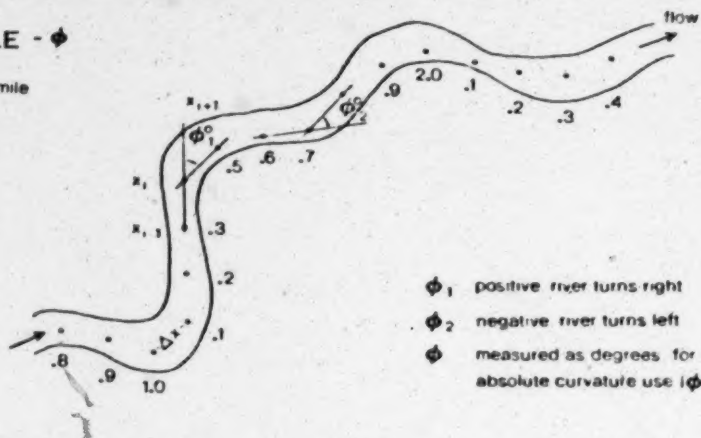


Figure 3.19 Research Council of Alberta jetboat -- series C -- at Kootenay River Bridge, beginning of survey (photo by R. Baldwin)

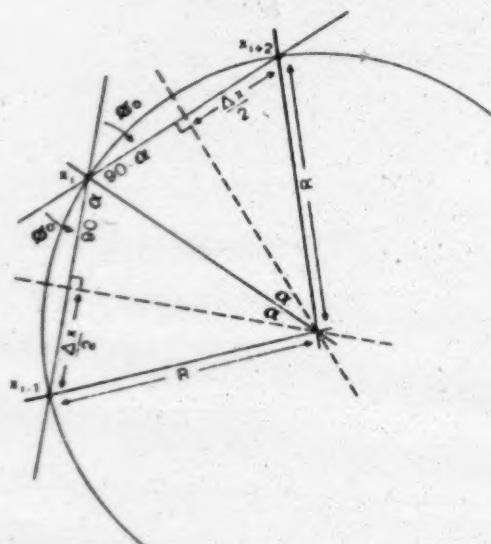
1) DEVIATION ANGLE - ϕ

0 .1 .2 .3 .4 .5 mile

Δx along path = .1 mile
(river distance in miles)

2) RADIUS OF CURVATURE - R

0 .1 mile



$$a) (90 - \alpha) + (90 - \alpha) + \phi = 180$$

$$\phi = 2\alpha$$

$$\frac{\phi}{2} = \alpha$$

$$b) \frac{\Delta x}{2} = \sin\left(\frac{\phi}{2}\right)$$

$$R = \frac{\frac{\Delta x}{2}}{\sin\left(\frac{\phi}{2}\right)}$$

$$c) R \text{ ft} = \frac{(.05 \text{ mi})(5280 \text{ ft mi}^{-1})}{\sin\left(\frac{\alpha \pi}{180}\right)}$$

assuming x_{i-1} , x_i , x_{i+1} lie on the circumference of a circle radius R

DETERMINATION OF ϕ AND R FOR PLANFORM

FIGURE 3.20

CHAPTER IV - ANALYSIS OF DATA

STATISTICAL PROPERTIES OF THE SERIES

A. STATIONARITY OF THE SERIES

The assumption of stationarity, at least in its weak sense, is necessary to analysis by variance spectrum statistics. This assumption, discussed earlier in Chapter 2, essentially provides that all properties of the series obtained remain constant throughout the ensemble. The sample thus shows independence from absolute time. A series that demonstrates trends in its means or variances is said to be non-stationary or evolutive.

Stationarity in the weak sense implies that the series may be characterized by its mean and variance, which do not show significant change over the length of the record. Assumptions of higher-order stationarity, in which higher order moments are also constant throughout the sample, are rarely found in nature.

Trends constituting non-stationarity of a time series may occur in the means or variances, or both. Simple filters quickly remove trends in the mean. However, trends in the variances along a series require more difficult manipulation. Deterministic elements must be calculated for the series and removed.

Downstream increase of stream discharge and river dimensions, especially present if the series contains major confluences, diminish the probability of stationarity for the series. In this study, both

the South Nahanni and Kootenay River reaches included confluences of unknown significance with respect to this property. The Beaver River reach contained no substantial confluence, although the river exhibited a very slight tendency to increase in width and depth downstream.

Each depth chart and planform series obtained for the study constituted a realization (sample function) of the stochastic process. Strict and even weak stationarity are statistical properties used to describe ensembles of time series. The only stationarity test appropriate for simple realizations of the ensemble is that of self-stationarity, which analyzes changes in properties along the sample series. A series is thus self-stationary if subseries derived from the main sample by simple segmentation demonstrate no significant changes in their moments when compared as units. Weak self-stationarity implies constancy, within random sampling fluctuations, of the first two moments. Yevjevich (1972) suggested that subseries with parameters confined within 95 per cent tolerance limits about the corresponding values of parameters for the entire realization, may be inferred to be self-stationary.

Table 4.01 presents chi-squared (χ^2) values obtained for tests of self-stationarity of depth variances and of both relative ($\frac{1}{2} \phi$) and absolute ($|\phi|$) curvature series. All series (A, B, C, and D) were tested and all were found to be weakly self-stationary.

Tests for self-stationarity of depths entailed initially the segmentation of each digitized corrected depth series into 100 blocks of equal sizes. Means and standard deviations were then calculated for each block, forming four new series (A, B, C, and D) consisting of one hundred block-mean depths each, and four new series (A, B, C, and D) consisting of one hundred block-standard deviations each. Frequency

Table 4.01: TESTS FOR SELF STATIONARITY OF RIVER DATA

Test Details	series A-S. Nahanni upstr.	series B-S. Nahanni dnstr.	series C Kootenay	series D Beaver R.	Comments
1. depths, first moment					
d.f.	7	7	7	7	self
χ^2 test value, 95%	14.067	14.067	14.067	14.067	stationarity
observed χ^2 statistic	7.1	1.5	8.7	10.13	upheld
2. depths, second moment					
d.f.	7	7	7	7	self
χ^2 test value, 95%	14.067	14.067	14.067	14.067	stationarity
observed χ^2 statistic	10.0	3.68	2.22	8.44	upheld
3. meanders, rel. curv. ($\frac{1}{\phi}$)					
d.f.	7	6	13	12	self
χ^2 test value, 95%	14.067	12.592	22.362	21.026	stationarity
observed χ^2 statistic	4.6	7.08	16.0	13.6	upheld
4. meanders, abs. curv. ($ \phi $)					
d.f.	4	4	7	7	self
χ^2 test value, 95%	9.488	9.488	14.067	14.067	stationarity
observed χ^2 statistic	1.39	3.04	13.2	11.13	upheld

histograms were constructed for each of these series. These are shown as figures 4.01, 4.02, 4.03, 4.04, 4.05, 4.06, 4.07, and 4.08. The frequencies found were used as expected values in the χ^2 tests performed later.

Each series was next considered as composite of two unit samples of equal lengths (50 blocks each) and these halves were then examined for similarity to each other and to their parent series. Histograms of each half-sample are included in figures 4.01 through 4.08. Frequencies found were considered observed frequencies for the χ^2 test. Expected frequencies were adjusted to correspond to the half-size of the samples tested, and χ^2 values were found in the usual manner, as

$$\chi^2 = \sum_{i=1}^n \frac{(e_i - o_i)^2}{e_i}$$

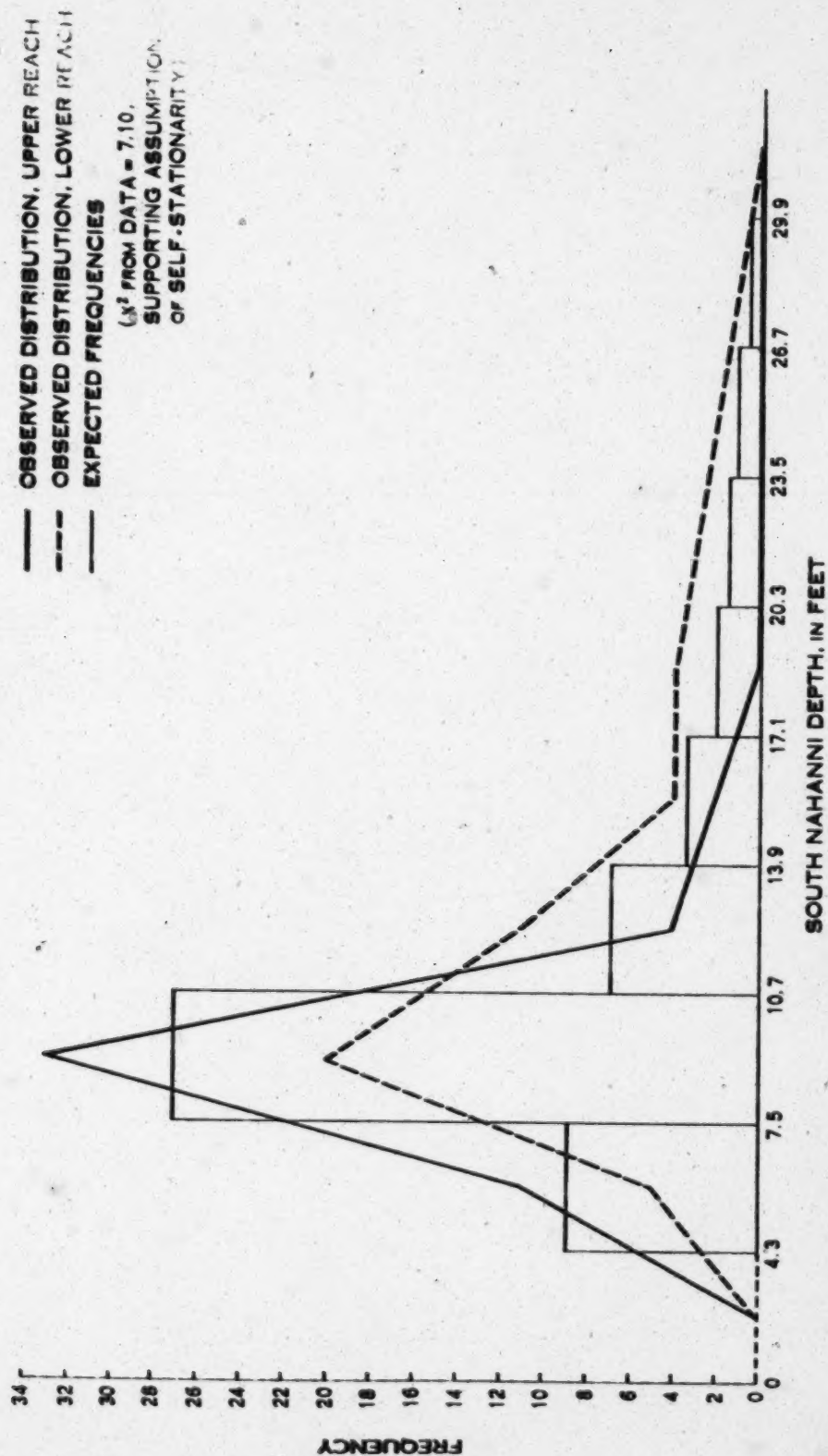
where e_i is the frequency expected, o_i is the frequency observed for each i 'th interval, and n specifies the number of frequency classes constructed for the test.

Statistical tables (Walpole, 1968) furnished critical test values at 95% tolerance and $\nu = n-1$ degrees of freedom. Degrees of freedom were calculated as $n-1$ since only the total number of observations was used in the calculation of expected values. The test thus ran:

$$H_0: \mu_1 = \mu_2 = \mu_s \quad \text{or} \quad \sigma_1 = \sigma_2 = \sigma_s$$

$$H_1: \mu_1 \neq \mu_2 \neq \mu_s \quad \text{or} \quad \sigma_1 \neq \sigma_2 \neq \sigma_s$$

where μ_1 and σ_1 are the mean and standard deviation of the first half of the parent series s , and μ_s and σ_s are the parameters of the parent series. A value of χ^2 greater than the value tolerated for 95% of occurrences at $n-1$ degrees of freedom would cause rejection of H_0 as beyond the limits of chance.



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF DEPTH SERIES MEANS — SOUTH NAHANNI UPSTREAM — SERIES A
 FIGURE 4.01



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY OF DEPTH SERIES,
 STANDARD DEVIATIONS - SOUTH NAHANNI UPSTREAM - SERIES A
 FIGURE 4.02

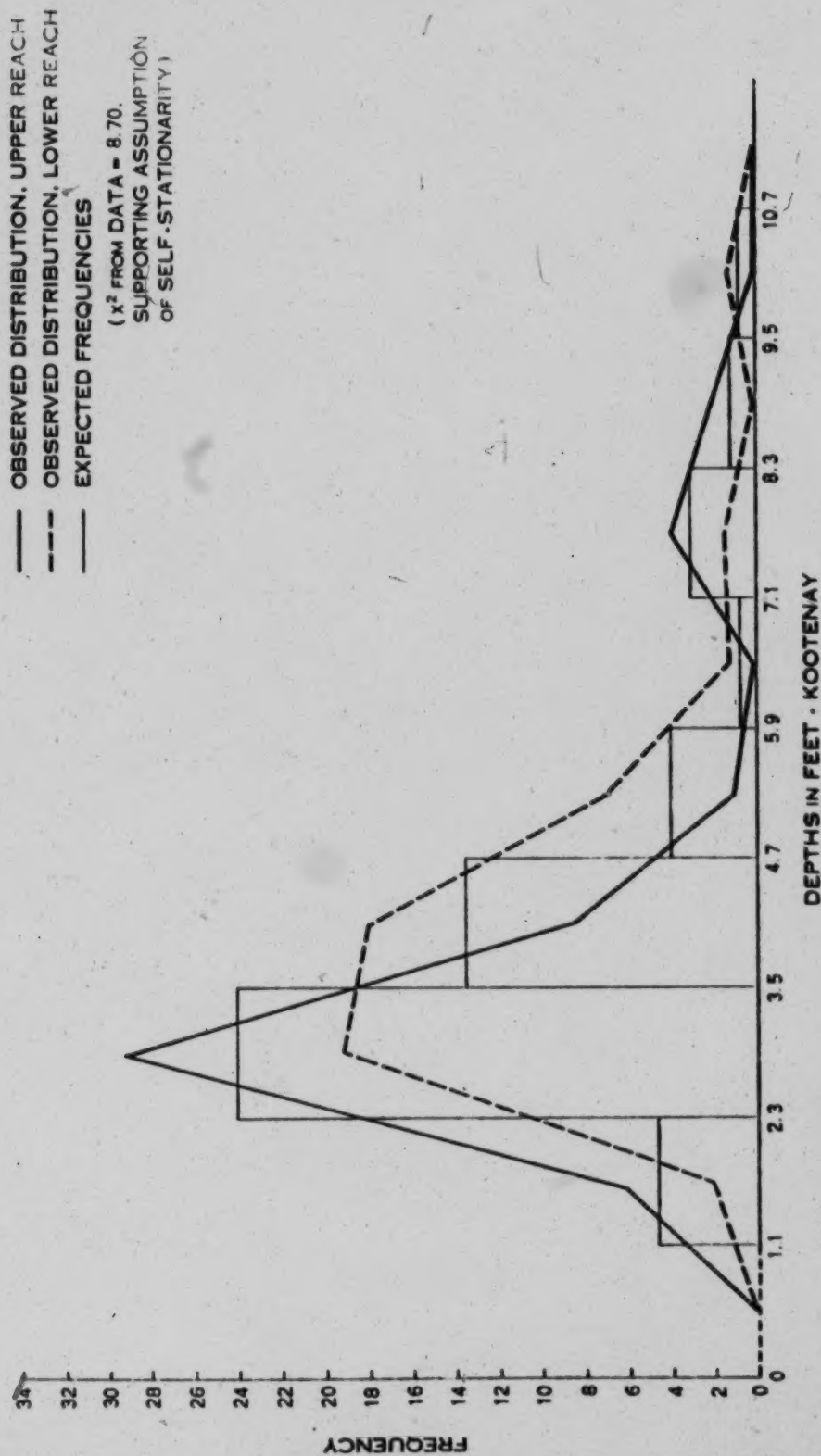


DEPTHS IN FEET - SOUTH NAHANNI B

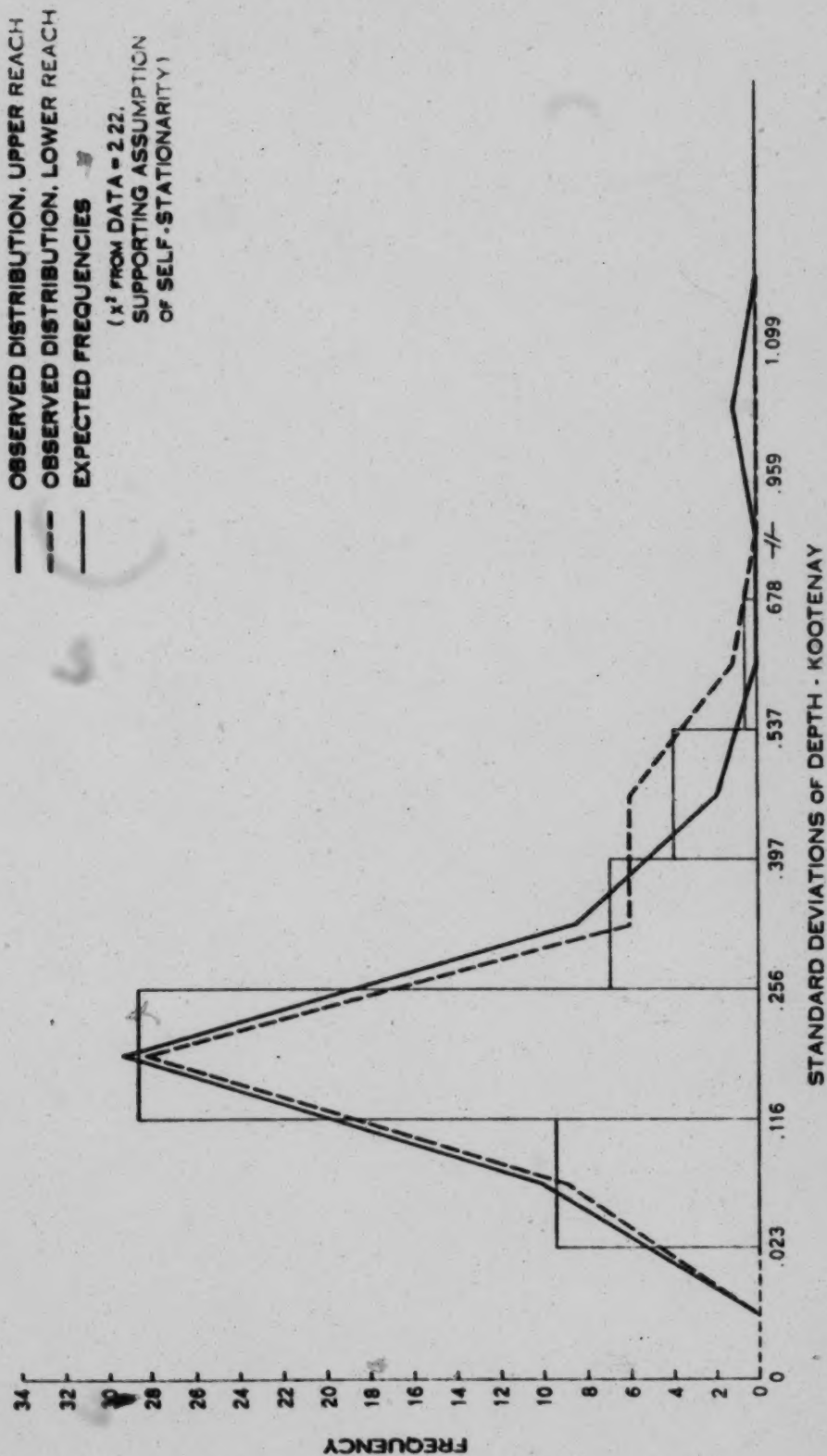
EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF DEPTH SERIES MEANS - SOUTH NAHANNI DOWNSTREAM - SERIES B
 FIGURE 4.03



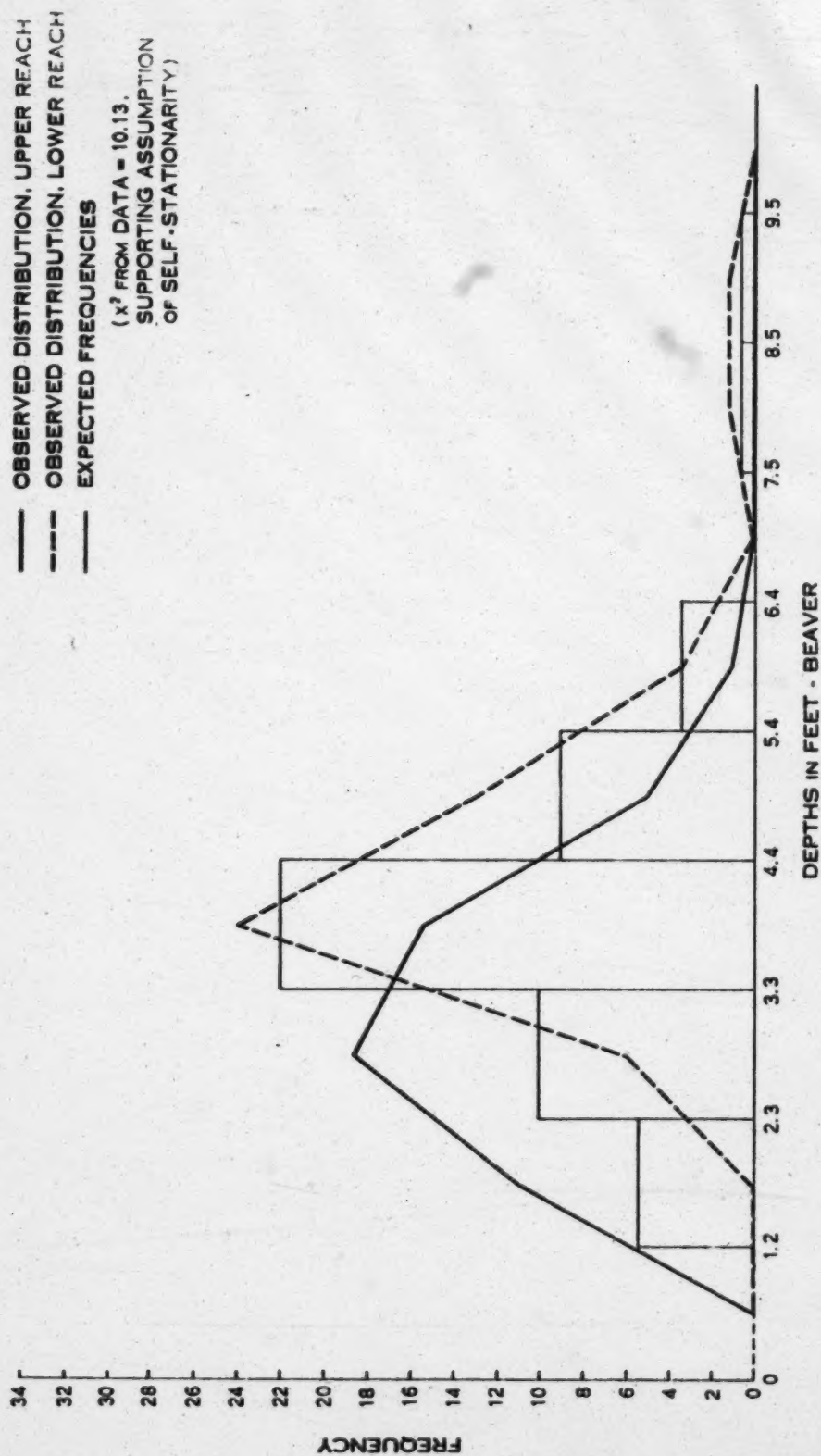
EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY OF DEPTH SERIES.
 STANDARD DEVIATIONS - SOUTH NAHANNI DOWNSTREAM - SERIES B
 FIGURE 4.04



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF DEPTH SERIES MEANS — KOOTENAY RIVER — SERIES C
 FIGURE 4.05

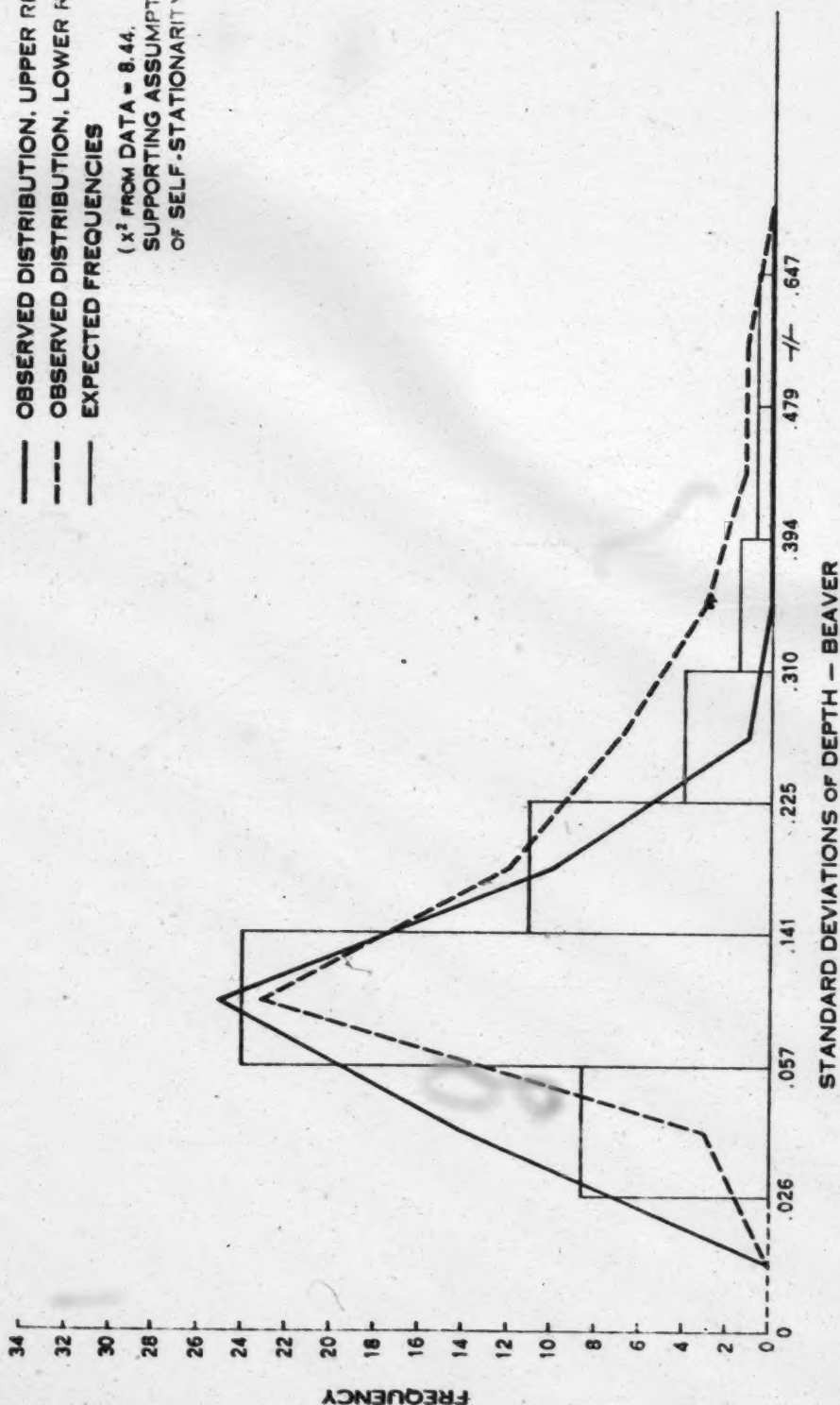


EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF DEPTH SERIES, STANDARD DEVIATIONS - KOOTENAY RIVER - SERIES C
 FIGURE 4.06



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF DEPTH SERIES MEANS - BEAVER RIVER - SERIES D
 FIGURE 4.07

— OBSERVED DISTRIBUTION, UPPER REACH
 - - - OBSERVED DISTRIBUTION, LOWER REACH
 — EXPECTED FREQUENCIES
 (χ^2 FROM DATA = 8.44,
 SUPPORTING ASSUMPTION
 OF SELF-STATIONARITY)



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF DEPTH SERIES, STANDARD DEVIATIONS - BEAVER RIVER - SERIES D
 FIGURE 4.08

All first and second order moments supported the assumption of self-stationarity of the depth series obtained, as seen in Table 4.01. All smaller reaches also exhibited weak self-stationarity at 95%.

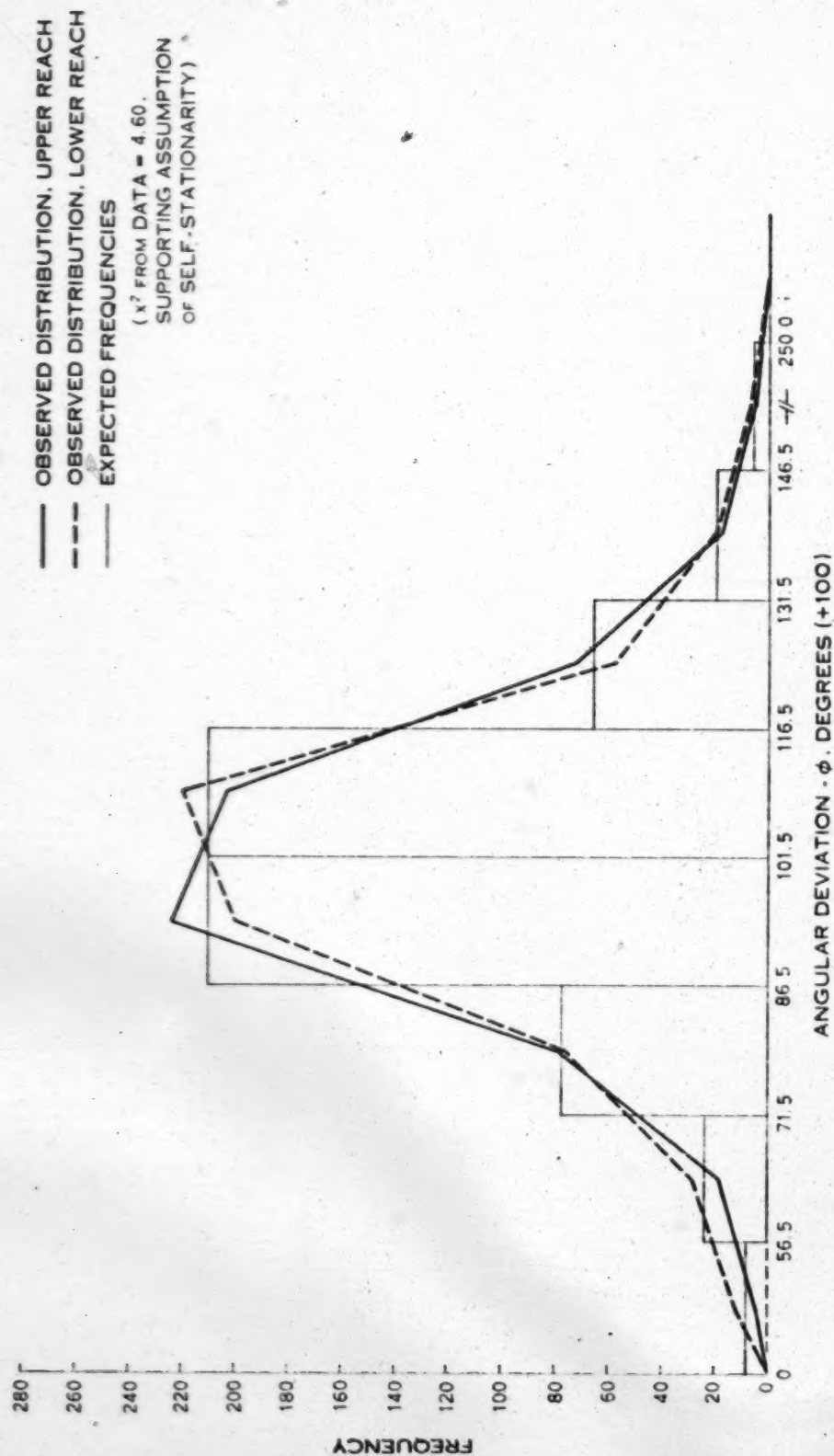
Meander data were also tested using χ^2 statistics. Each observation was recorded at $\Delta x = 0.1$ miles, and for the χ^2 test, it was assumed that each angular derivation was thus a mean for 0.1 miles of river path. Circular paths were assumed on the micro-scale used in the calculation of radius of curvature, and so variances of angular deviation fluctuations, with the 0.1 mile path, were assumed constant. Both relative and absolute curvature series were tested in their "means", then, and all were found to be well within tolerance limits of 95%. Meander planform series were thus also found to be weakly self-stationary, as indicated in Table 4.01. Figures 4.09 through 4.16 depict histograms of the meander planform data.

Results obtained from spectral and cross-spectral analyses of of the data may thus be treated with greater confidence under the assumption of self-stationarity. The data thus allow valid use of the statistical technique chosen.

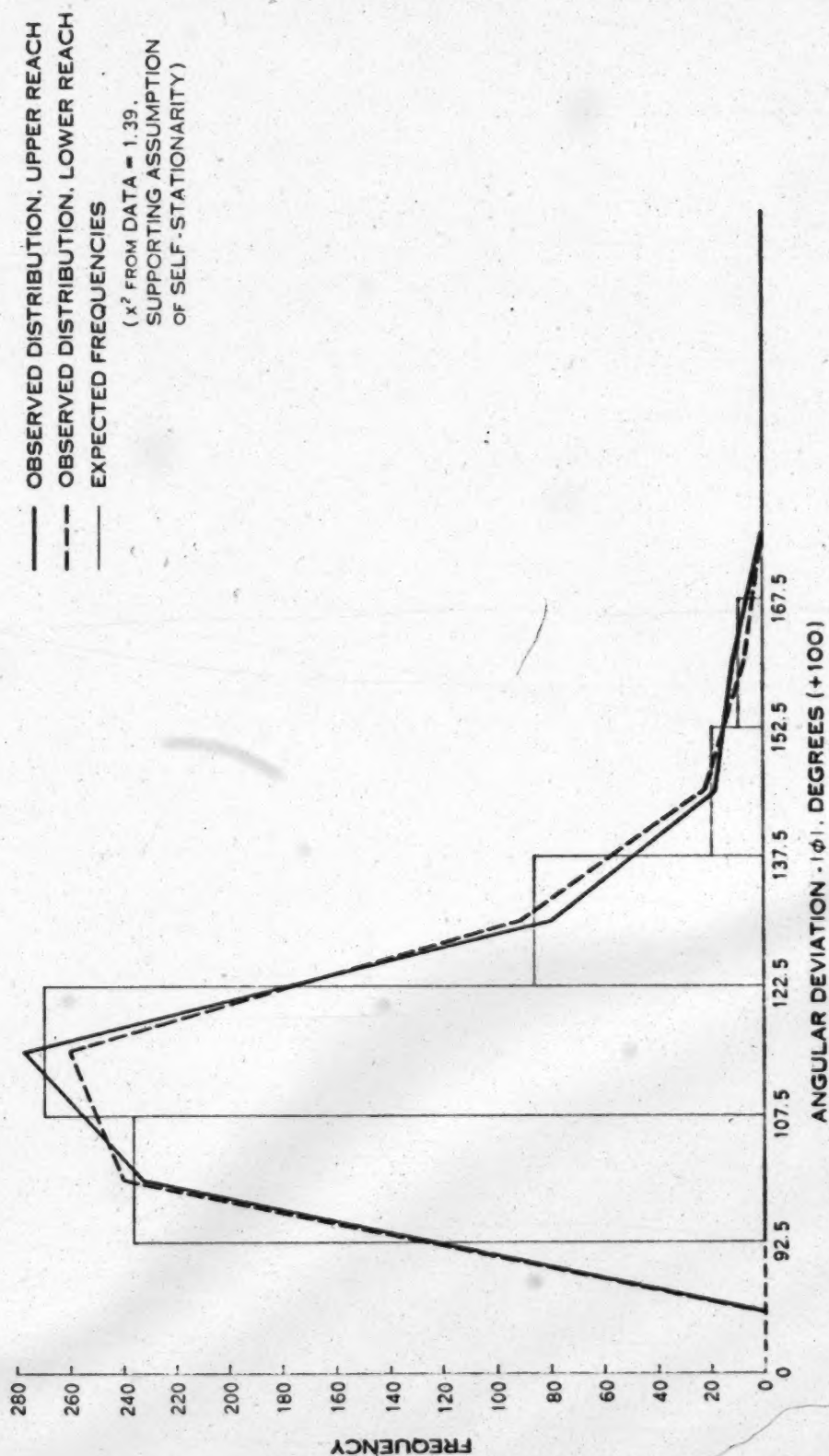
B. INFORMATION CONTAINED IN THE RADIUS OF CURVATURE

Radius of curvature was calculated, as shown in Figure 3.14, for all deviation angles measured for the planform analysis of the rivers studied. Calculation of this statistic was based on the assumption that for any three adjacent points in the planform series, a circle fit to the points reasonably approximated the flow of the river. Figure 4.17 presents graphically the radius of curvature statistics for the study.

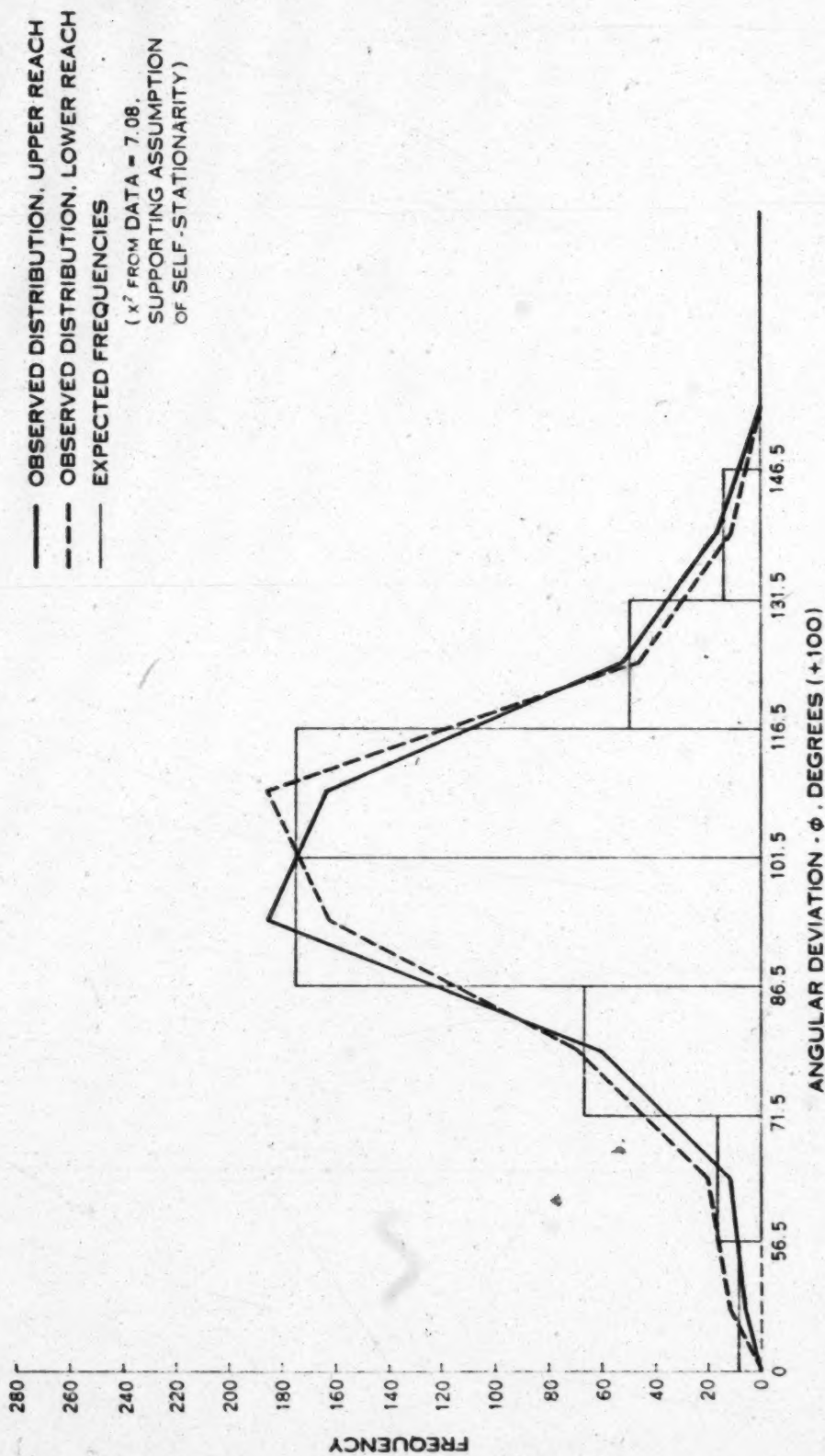
Radius of curvature was chosen in order to examine preferred



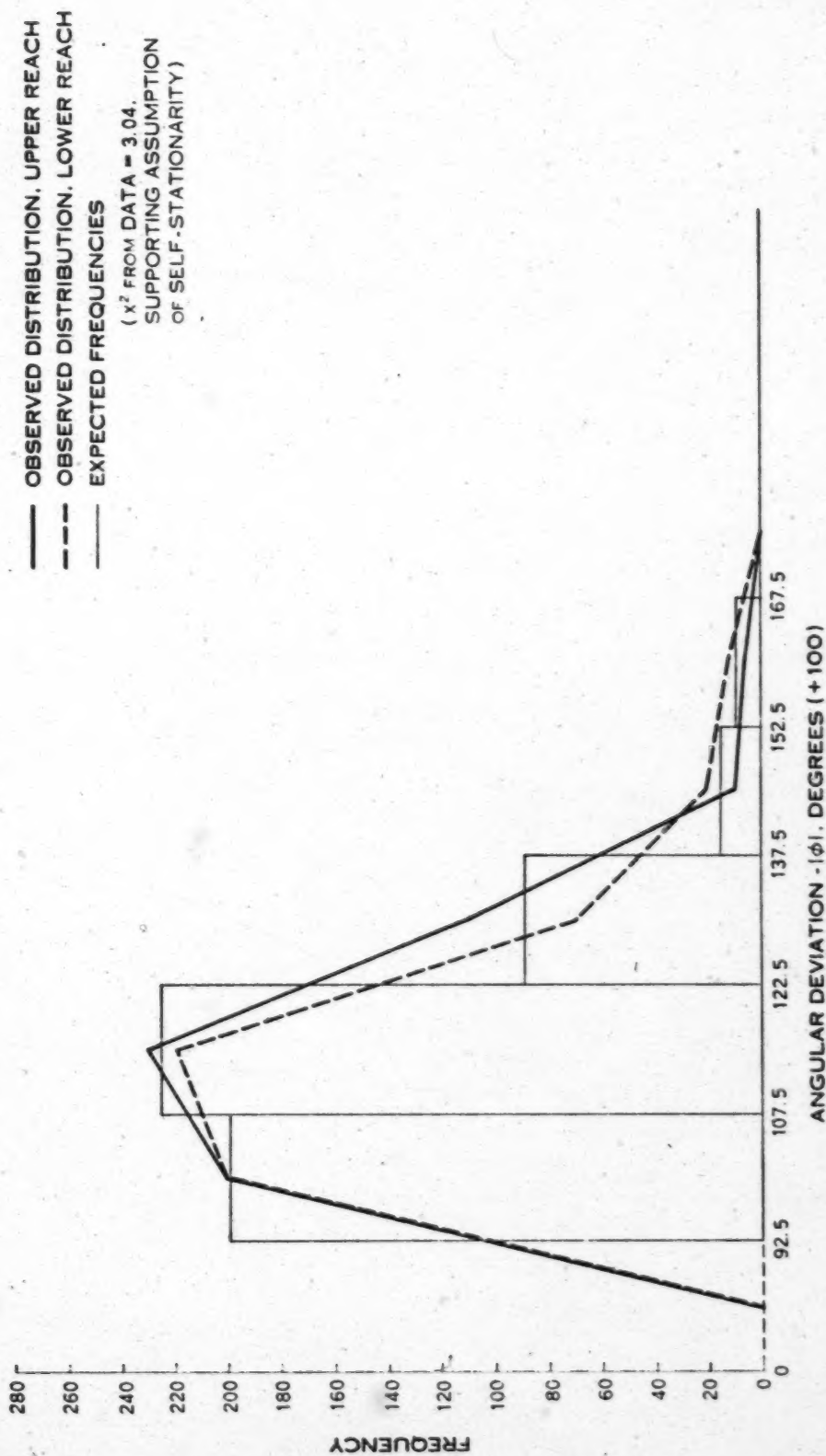
EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF RELATIVE CURVATURE (PLAN) FOR SERIES A, SOUTH NAHANNI UPSTREAM
 FIGURE 4.09



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF ABSOLUTE CURVATURE (PLAN) FOR SERIES A, SOUTH NAHANNI UPSTREAM
 FIGURE 4.10

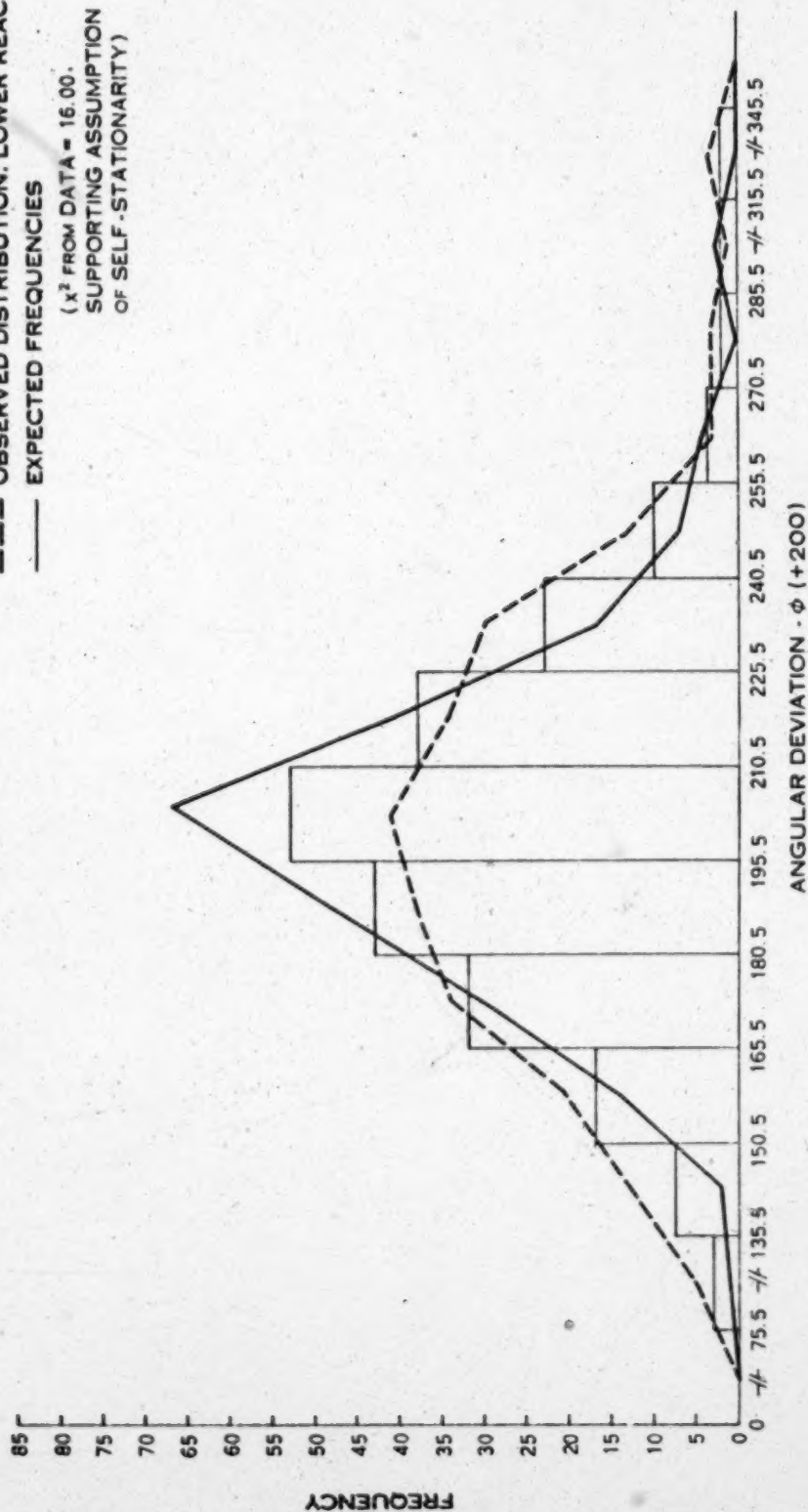


EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF RELATIVE CURVATURE (PLAN) FOR SERIES B, SOUTH NAHANNI DOWNSTREAM
 FIGURE 4.11

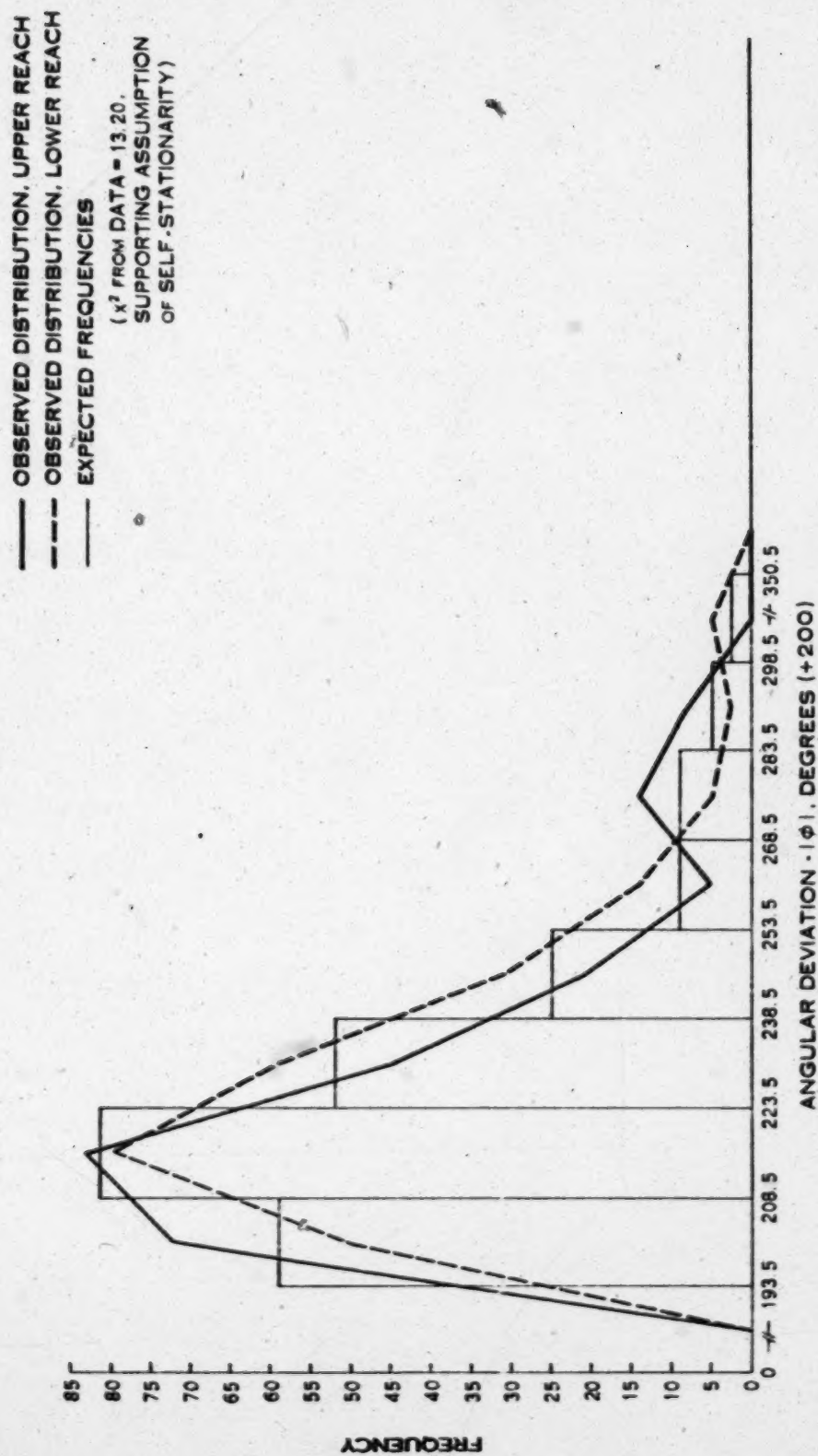


EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF ABSOLUTE CURVATURE (PLAN) FOR SERIES B, SOUTH NAHANNI DOWNSTREAM
 FIGURE 4.12

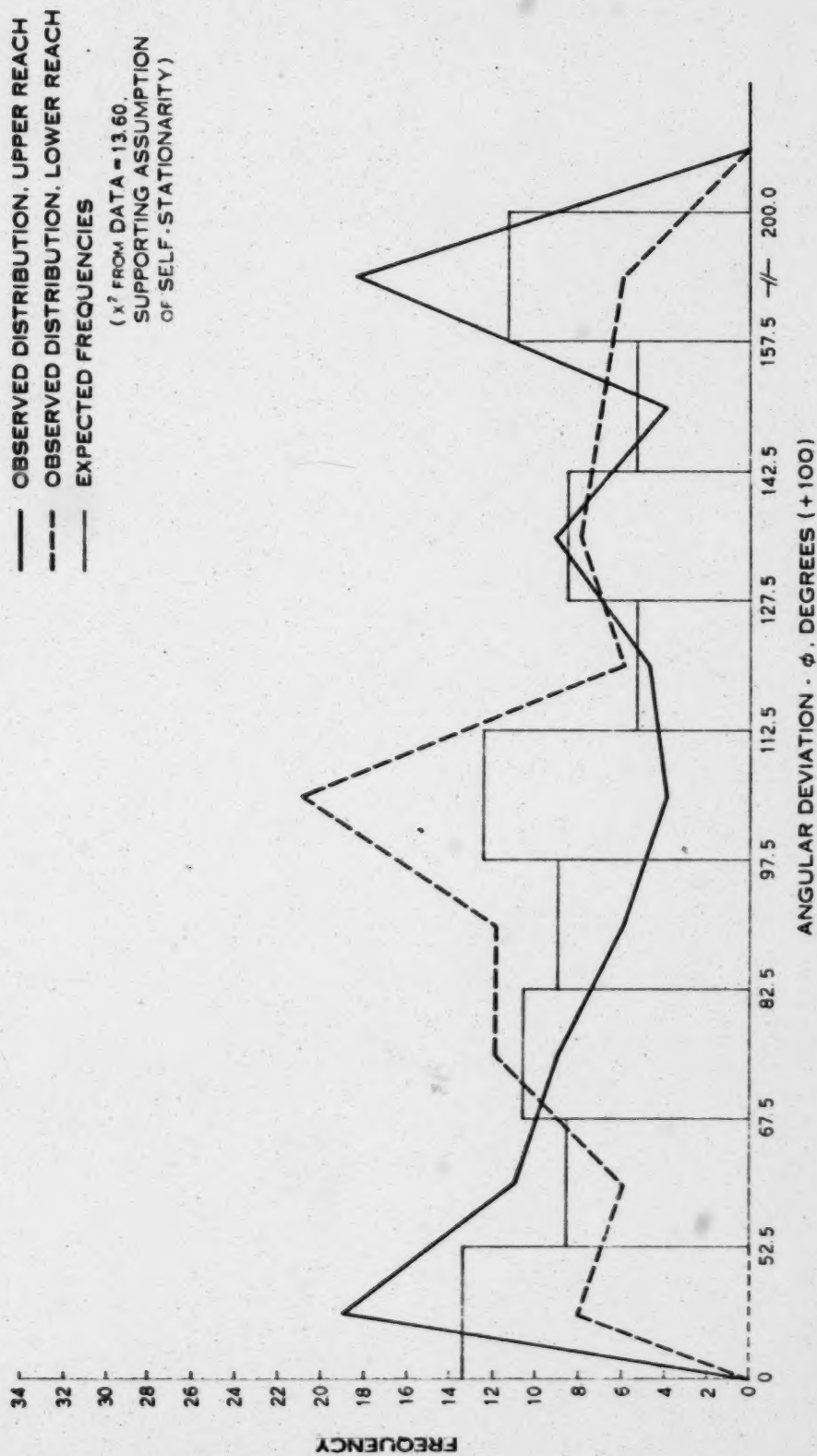
— OBSERVED DISTRIBUTION, UPPER REACH
 - - - OBSERVED DISTRIBUTION, LOWER REACH
 — EXPECTED FREQUENCIES
 (χ^2 FROM DATA = 16.00.
 SUPPORTING ASSUMPTION
 OF SELF-STATIONARITY)



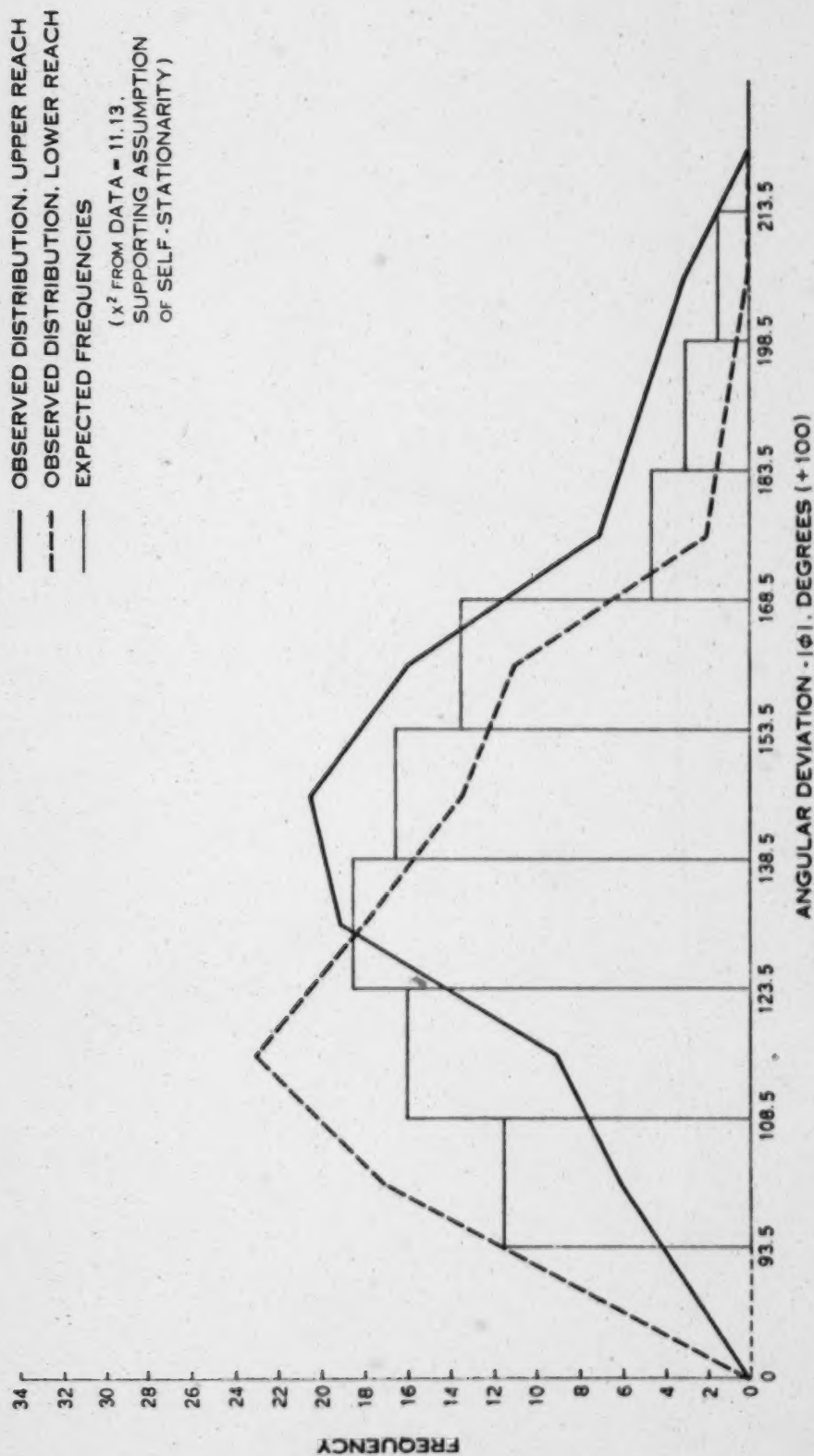
EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF RELATIVE CURVATURE (PLAN) FOR SERIES C — KOOTENAY RIVER UPSTREAM
 FIGURE 4.13



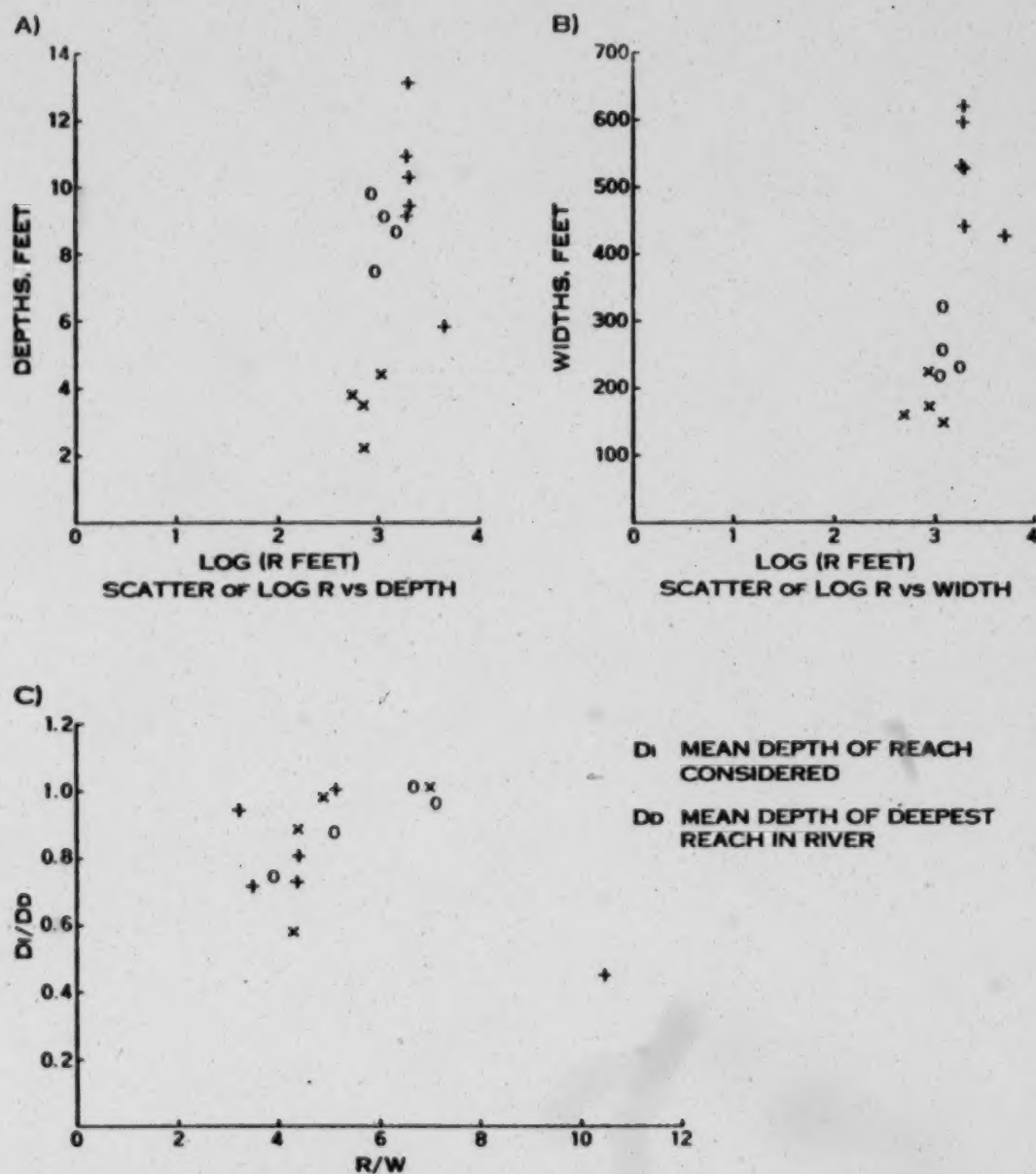
EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF ABSOLUTE CURVATURE (PLAN) FOR SERIES C — KOOTENAY RIVER UPSTREAM
 FIGURE 4.14



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF RELATIVE CURVATURE (PLAN) FOR SERIES D — BEAVER RIVER UPSTREAM
 FIGURE 4.15



EXPECTED AND OBSERVED FREQUENCY DISTRIBUTIONS
 FOR χ^2 TEST FOR SELF-STATIONARITY
 OF ABSOLUTE CURVATURE (PLAN) FOR SERIES D — BEAVER RIVER UPSTREAM
 FIGURE 4.16



**RADIUS OF CURVATURE RELATIONS — GRAPHED vs DEPTH
 A), WIDTH B), AND GRAPHED AS R/W vs DEPTHS C)
 FIGURE 4.17**

angular deviations, as given in table 4.02, in non-dimensional terms. Radius of curvature was divided by river width for each reach to provide a dimensionless value for study. It was found that for reaches with meanders or curves, the parameter R/W clustered in a small range of values. River size or scale factors thus seemed eliminated when the statistic was used.

The parameter R/W showed an interesting tendency to approach the value 27 as bend tightness increased (ie. as the sinusoidal form was approached). The significance of this observation is difficult to ascertain in light of the "approximate" nature of the values used.

Figure 4.17 presents observations on the statistic R/W . It shows no distinct relationships with width or depth of the rivers, but size of sample (small) and accuracy in deviation of R/W make inferences difficult. The figure is included in order only to present the data as it was found.

The values presented can only be regarded as rough approximations of their true values. Widths were obtained from maps and air photographs rather than in the field, and R was calculated for angular deviations accurate to one degree. Inaccuracy involved thus in the computation of R prevented its use in more sophisticated analysis, but it seems a good parameter, as it directly relates to flow acceleration in bends (centrifugal acceleration, a , where $a = v^2/R$, and v is the velocity of flow.) Further work with the parameter is planned, but remains beyond the scope of this study.

Table 4.02: RADIUS OF CURVATURE NON-DIMENSIONAL STATISTICS: $\frac{R}{W}, \frac{R}{\pi W}$

A. South Nahanni R.	mean radius of curvature, ft.	log R (approx.)	river width, ft.	$\frac{R}{W}$	$\frac{R}{\pi W}$	Comments
1. main braids $\pm \phi$	∞ to 30253		612	49.4	15.7	1. measurement
$ \phi $	2023	3.3	612	3.3	<u>1.1</u>	of w not good.
2. first canyon $\pm \phi$	∞ to 30253		438	69.1	22	for braided
$ \phi $	2332	3.4	438	5.3	<u>1.7</u>	sections, based
3. intermediate braids $\pm \phi$	∞ to 30253		597	50.7	16.1	on width of
$ \phi $	2332	3.4	597	3.9	<u>1.2</u>	main channel
4. Second Canyon $\pm \phi$	∞ to 30253		533	56.8	18.1	2. absolute
$ \phi $	2332	3.4	533	4.4	<u>1.4</u>	curvature values
5. Third Canyon $\pm \phi$	∞ to 30253		533	56.8	18.1	$\frac{R}{\pi W}$ underlined
$ \phi $	2332	3.4	533	4.4	<u>1.4</u>	because, since
6. Fourth Canyon $\pm \phi$	6052		422	14.3	4.6	direction of
$ \phi $	4324	3.6	422	10.2	3.4	curve not

relevant to

calculation of

R, these are the

focus of the study.

		mean radius of curvature, ft.	log R (approx.)	river width, ft.	$\frac{R}{W}$	$\frac{R}{\eta W}$	Comments
B. Kootenay River							
1. braids $\pm \phi$		∞ to 30253		317	95.5	30.4	
	$ \phi $	1174	3.07	317	3.70	<u>1.2</u>	
2. lower semi-	$\pm \phi$	∞ to 30253		211	143.2	45.6	
entrenched $ \phi $		1384	3.14	211	6.65	<u>2.1</u>	
3. upper reach $\pm \phi$		∞ to 30253		224	134.8	42.9	
	$ \phi $	1600	3.20	224	7.1	<u>2.3</u>	
4. entire reach $\pm \phi$		∞ to 30253		251	120.6	38.4	
	$ \phi $	1324	3.12	251	5.3	<u>1.7</u>	
C. Beaver River							
1. downstream $\pm \phi$		∞ to 30253		158	191	61.0	
meanders $ \phi $		1091	3.03	158	6.9	<u>2.2</u>	
2. straight $\pm \phi$		∞ to 10085		164	61.6	19.6	
	$ \phi $	602	2.77	164	3.7	<u>1.2</u>	
3. upstream $\pm \phi$		∞ to 15127		211	71.6	22.8	
	$ \phi $	878	2.94	211	4.2	<u>1.3</u>	
4. entire reach $\pm \phi$		∞ to 30253		180	168.5	53.6	
	$ \phi $	832	2.92	180	4.6	<u>1.5</u>	

Table 4.03: PREFERRED ANGULAR DEVIATIONS AND CORRESPONDING VALUES OF R FOR REACHES IN STUDY

A. South Nahanni River (series A and B)	relative curvature		absolute curvature		comments
	$\bar{\theta}^\circ$	σ range	$\bar{\theta}^\circ$	σ range	
1. main braids	100.2	20.5 160	114.5	13.8 86	arbitrary mean of
2. First Canyon	100.7	17.2 122	112.8	11.5 63	100° chosen for
3. intermediate braids	100.0	16.4 97	113.5	11.5 47	convenience of
4. Second-Third Canyons	100.2	16.4 108	112.7	10.3 56	calculation for
5. Fourth Canyon	94.7	11.2 52	106.9	5.3 23	South Nahanni R.,
6. entire river reach	100.2	18.0 169	113.4	12.0 96	measurements taken in
					downstream direction
7. radius of curvature:	angle ^o -	-7° -6° 0	+1° +6° +12° +13° +14° 15°		
	R ft. -	4324 5044	30253 5044 2526 2332 2166 2033		

observation: bends become tighter downstream, including braided sections

tendency for mean to become zero (balance of reversals of curvature) present

Table 4.03: Continued

B. Kootenay River

(series C)

	relative curvature		absolute curvature		comments
	$\bar{p}$	σ	$\bar{p}$	σ	
1. braids	200.3	31.3	117	58	arbitrary mean of 200° chosen for convenience of calculation
2. lower semi-entrenched (below White R.) reach	200.1	38.6	250	132	
3. upper entrenched reach (above White R.)	200.4	25.9	180	95	for Kootenay River, measurements taken in upstream direction
4. entire reach	200.2	31.2	250	132	

5. radius of curvature:	angle° -	+1	19	20	22	23	24	26
	R, ft. -	30253	1600	1520	1384	1324	1270	1174

observations: bends become tighter downstream, including braids

tendency for mean to become zero (balance of reversals of curvature) present

Table 4.03: Continued

C. Beaver River

(series D)

	relative curvature		absolute curvature		comments
	$\bar{\gamma}^\circ$	δ range	$\bar{\gamma}^\circ$	δ range	
1. downstream meanders	100.3	35.7 158	128.5	21.3 84	arbitrary mean of 100° chosen for
2. straight section	97.6	40.6 137	135.4	19.1 75	convenience of
3. upstream meanders	97.0	58.5 200	152.0	26.6 98	calculation
4. entire reach	98.7	44.3 200	136.9	24.6 99	for Beaver River, measurements taken in upstream direction

5. radius of curvature:	angle ^o	-4	-3	-2	-1	0	+1	28	37	35	52
	R, ft.	7565	10085	15127	30253		30253	1091	832	878	602

observations: bends become less tight downstream - small sample

tendency towards preference of left turns - trend induced by larger channel in which Beaver

River is misfit

C. INFORMATION CONTAINED IN SIMPLE FIRST AND SECOND MOMENT STATISTICS

Simple first and second moment statistics can be used to describe a limited range of form-process phenomena related to both the bed configuration and the planform geometry of rivers. The mean and standard deviation may be useful in distinguishing reaches within a longer series describing the river, where certain reaches are comprised of a number or blocks, the characteristics of which are similar within each reach, but significantly different between reaches. The information contained in the means and standard deviations of the depth and angular change series provides a general overview of changes characteristic of the river's behavior along the path of flow.

1. Depth Series

a) General:

Means and standard deviations were computed for individual blocks in each "corrected" depth series (chapter 3 explains the correction performed on the raw data). Blocks were then arranged according to the different geographic reaches contained within the whole series, and mean depths and average deviations were calculated for each reach. Table 4.04 presents these statistics (first and second moments) for all the rivers.

A strong pattern is evident in this data. Straight sections of channel are characteristically shallow relative to the river's mean depth, and when not entrenched, they exhibit substantially smaller depth variation along the flow path than other reaches. Sections with bends exhibit greater mean depths and depth variations, and regularly meandering sections show both the greatest mean depths and the greatest variations

Table 4.04:A FIRST AND SECOND MOMENTS OF DEPTHS FOR REACHES STUDIED
 FOR SOUTH NAHANNI RIVER - includes both series A - upstream,
 and series B - downstream

Reach	Depth Statistics in feet		Comments
	$\bar{x}$	s.d.	
1. Series A - entire	10.82	5.59	
main braids (mi. 81-115)	12.55	6.98	A: greatest variation
First Canyon (mi. 68-81)	12.99	4.63	A: most deep
intermediate braids (mi. 59-68)	9.12	4.31	
Second Canyon (mi. 49-59)	9.29	3.32	
Third Canyon (mi. 27.5-49)	10.33	4.20	
flats (mi. 7-27.5)	8.83	3.86	
Fourth Canyon (mi. 4.5-7)	5.62	1.74	A: most shallow A: least variation
2. Series B - entire*	11.74	5.11	
main braids	11.21	5.79	B: greatest variation
First Canyon	12.49	4.82	
intermediate braids	10.08	4.13	B: most shallow
Second Canyon	12.94	3.42	
Third Canyon	13.01	5.04	B: most deep
flats	10.18	2.77	B: least variation

*note, series B excludes Fourth Canyon and half the flats section, accounting for discrepancy with series A.

Table 4.04:B FIRST AND SECOND MOMENTS OF DEPTHS FOR ALL REACHES STUDIED
FOR BOTH SERIES C - KOOTENAY RIVER AND SERIES D - BEAVER
RIVER

Reach	Depth Statistics in feet		Comments
	$\bar{x}$	s.d.	
3. Series C - entire	8.50	2.76	
braids (mi. 1-5.5)	7.50	2.35	C: most shallow
semi-entrenched (mi. 5.5-20.5)	9.65	2.77	
straights section (mi. 20.5-25.5)	8.17	1.84	C: least variation
irregular meanders (mi. 25.5-26.5)	8.74	3.65	
rock canyon (mi. 26.5-29)	11.28	4.01	C: most deep C: most variation
shallow-incised irregular meanders (mi. 29-47.1)	7.75	2.33	
4. Series D - entire	3.77	1.80	
meanders - border (mi. 1-9)	4.27	2.06	D: most deep D: greatest variation
gentle curves (mi. 9-9.7)	3.61	1.63	
straight (mi. 9.7-13)	2.44	0.82	D: most shallow D: least variation
meanders (mi. 13-18.25)	3.96	1.38	

in depth. In general, for non-incised reaches, depth variation tends to increase as tightness of bend increases (ie. as radius of curvature decreases), implying a relationship of greater variation in bed configuration for reaches of greater variation in angular deviation. Table 4.05 lists the rank order of depth variations and the corresponding rank order of angular deviation variations. Braided sections generally show greater depth variations than either the straight or irregularly meandering reaches, though their mean depths are less-than-average in the context of the entire river. Entrenched sections, where flow is confined between rock banks, as in the cases of First, Second, and Third Canyons of the South Nahanni, and the upper reaches of the Kootenay, also show both large variations in depth along channel and deep channels relative to other reaches in the same river.

b) Beaver River:

The portion of the Beaver River studied for this paper consists of two separate reaches of regular meanders, misfit within a larger valley, joined by more gentle curves of channel which continue to become a rather straight path of flow. Figures 3.06 and 3.07 include air photographs, actual depth chart, and depths corrected for boat velocity variations from these reaches. The meandering reaches exhibit greater depth variations than either the straight section or the sections of gentle curves, and the mean depths are correspondingly greater for the meandering reaches. The reaches of gentle curves are both deeper and more variable in their depths than the straighter reach. Downstream meanders tend to be generally deeper than their upstream counterparts.

Variations in the depth profile of the meandering reaches tend

Table 4.05: RANK ORDERING OF VARIATIONS IN ϕ AND DEPTH FOR ALL REACHES
(greater variance assigned lower magnitude rank)

A. Series A and B, South Nahanni

Reach	A $ \phi $	A $^+\phi$	A depths	B $ \phi $	B $^+\phi$	B depths
1. Main braids	1	1	1	1	1	1
2. First Canyon	3	3	2	3	3	3
3. intermediate braids	2	2	3	2	2	4
4. Second Canyon	4	4	6	4	4	5
5. Third Canyon	4	4	4	4	4	2
6. flats	-	-	5	-	-	6
7. Fourth Canyon	5	5	7	5	5	-

B. Series C

Reach	C $ \phi $	C $^+\phi$	C depths
1. braids	3	2	4
2. below White River	1	1	3
3. above White River:			
- straight section	2	3	6
- irregular meanders			2
- rock canyon			1
- irregular meanders			5

C. Series D

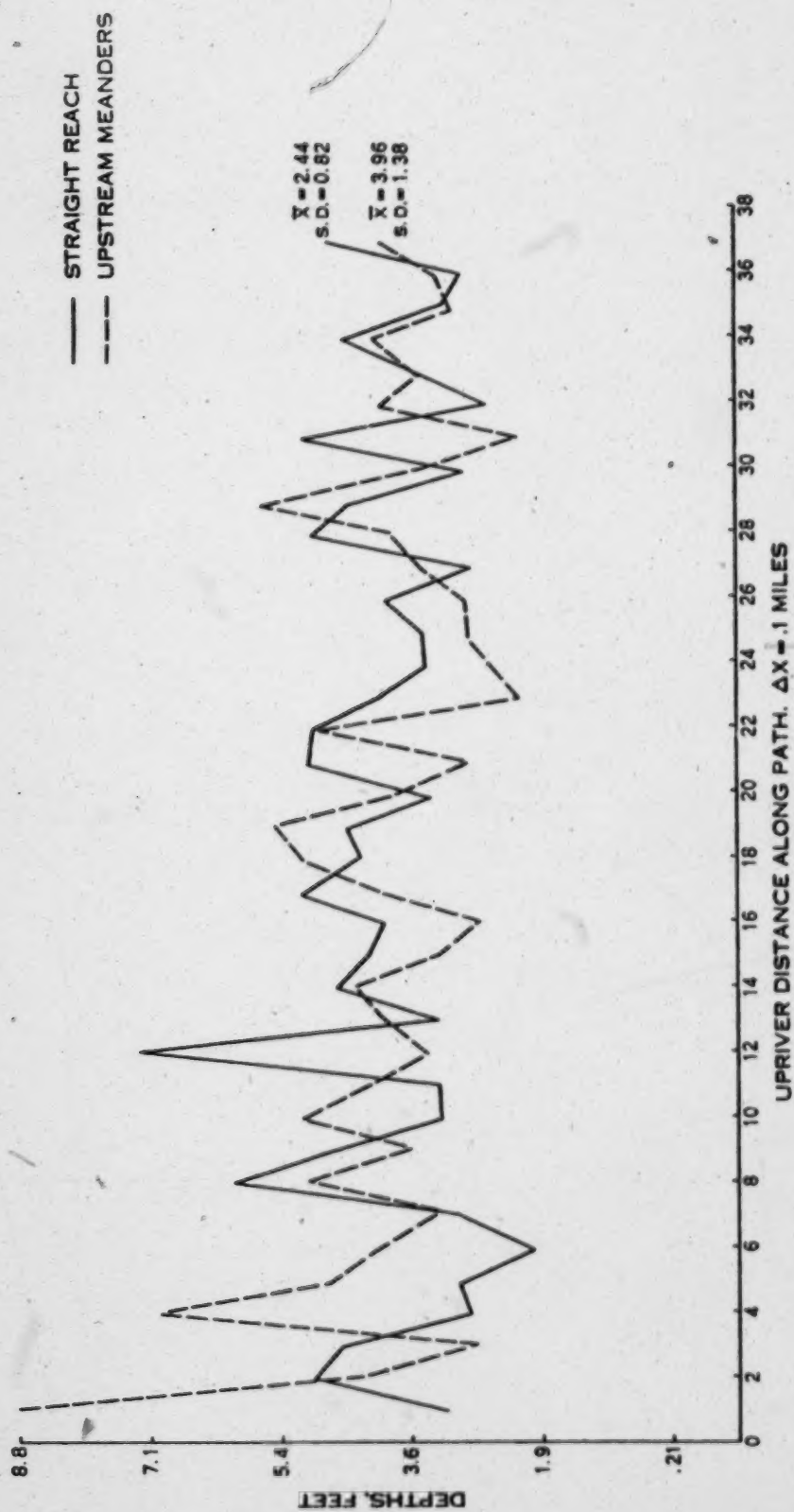
Reach	D $ \phi $	D $^+\phi$	D depths
1. downstream meanders	2	3	1
2. gentle curves	3	2	2
3. straight section			3
4. upstream meanders	1	1	4

to be explained more by a regularly spaced pattern of well-separated pools and shallows, as may be seen on the depth chart in Figure 3.06, than by smaller scale, higher frequency bed undulations. Pools occur regularly at the sharpest portions of bends along the meandering sections, while shallows are more pronounced at points of inflection. In contrast, the depth series of the straighter section consists of higher frequency, lower amplitude oscillations. The Beaver is a sand bed river, and the variance in the straight section is explained, at least in part, by the occurrence of dunes. The air photo in Figure 3.07 shows that dunes characterize the bed in straight portions of channel. Figure 4.18 allows quick comparison of the actual depth series for the contrasting sections, while listing corresponding depth statistics.

c) Kootenay River:

The portion of the Kootenay River studied for this paper begins in braids, but upstream the river incised into irregularly meandering sections broken by straight sections. The two main straight sections provide a good comparison of bed configuration, as one section is an incised rock canyon while the other is confined by a high rock wall on its left bank, but flows past a lower terraced right bank. The rock canyon was both the deepest and most depth-variable reach, while the semi-confined straight reach was among the most shallow and least depth variable for the river. The irregularly meandering sections were generally deeper than the river's average flow depth, and they showed more pronounced pool-and-riffle sequence in the river series. The braids form the shallowest section of river.

The highest frequency of depth fluctuation (in long profile)



ACTUAL DEPTH SERIES, WITH SAMPLING AT $\Delta X = .1$ MILES, COMPARING STRAIGHT AND MEANDERING REACHES IN THEIR BED CONFIGURATIONS, BEAVER RIVER
 FIGURE 4.18

occurs in the rock canyon, shown in Figure 3.09, where fluctuations are also of great amplitude. Flow is most turbulent in this section, where four-foot standing waves made upstream progress difficult even by jet-boat. Not far downstream, the less entrenched straight reach depth profile exhibits a lower frequency, lower amplitude series of depth fluctuations. Flow was less turbulent, and the river was wider here than where it became more entrenched upstream.

Upstream from the rock canyon, the river remains entrenched, though not as deeply, and it begins a pattern of irregular meandering. Depths in this section, according to the profile obtained, vary with high amplitude and intermediate frequency. The deepest pool (44 feet or about 14 metres) was recorded in this section, while pools here averaged a depth of fifteen to twenty feet. The width was confined, though not canyoned, and flow was again rather rough, characterized by standing waves.

The braided section begins where the Kootenay emerges from entrenchment and flows through a broad valley. Depths in this braided section are generally less than the river's mean depth, and bed undulations occur irregularly in the bed.

d) The South Nahanni River:

The South Nahanni River, below Virginia Falls, consists of four deeply entrenched reaches joined by braided channels, finally opening into intense braiding that continues to the Liard confluence. Three of the canyons meander irregularly, while one is comprised of rather regular meanders (comparison provided in Figures 3.10 and 3.11). First Canyon, the most regularly meandering and most deeply entrenched section, exhibits

much greater-than-average depth and depth variation in the river series. The less regularly meandering Second and Third Canyons also possess very deep channels and highly depth-variable bed configurations. Third Canyon, in which The Gate is located (shown in Figure 3.13), shows both the greatest mean depth and greatest depth variation for the river. The braided sections show slightly less depth and depth variance, but are deeper and more variable in their depths than the irregularly meandering Fourth Canyon just below Virginia Falls. Fourth Canyon is the most shallow and least depth variable section among the reaches studied. It is the farthest upstream reach, and thus directs the smallest volume of water.

First, Second, and Third Canyons vary in their depth series with large amplitude oscillations related strongly to the occurrence of bends. Sharper bends show deeper pools and again, the "shallows" (here, about ten to fifteen feet in depth) occur regularly at points of inflection. Higher frequency, lower amplitude fluctuations appear in the series, but are noise and their peaks are not particularly related to position in the bends. Variance of the series is mainly controlled by a pool-and-riffle sequence.

Braided reaches between canyons exhibit higher frequency, lower amplitude depth fluctuations, and show slightly higher depth variances, but smaller values for mean depth. The braids below the canyons approximate the rivers mean depth. This reach is shallow relative to the canyons, and marginally less variable in depth. Undulations of intermediate frequency and amplitude show only little relationship to planform within the noisy data array.

c) Theoretical Implications of Depth Observations, Using Mean and Variance

Observations based on the first two moments of the depth series data gathered for this study support the theories of Langbein and Leopold (1966) and Leopold and other (1960) with respect to differences in bed configurations of curved and straight reaches. Data discussed here was obtained for rivers of small, intermediate, and large sizes, and a variety of channel patterns and bed materials were considered, though briefly.

A pattern of increasing mean depth and depth variance with increasing regularity of meandering and increasing intensity of entrenchment is definitely established within the depth data. The increasing depth and depth variance patterns shown are consistent with the theories of equalization of energy dissipation through unit lengths of river, which propose that curves introduce additional organization into the distribution of channel depth, providing an extra form of energy distribution not available in straighter reaches. This form of increased entropy is assumed to be significant in the stabilization of the meandering form, thus accounting for the meandering behavior of rivers (Leopold and Langbein, 1962).

2. Planform Series

a) General

Mean angular deviation and variance of angular deviations were derived for all reaches of all rivers studied. Table 4.03 presents first and second order moments for all the reaches. It is interesting to note that most reaches and most rivers, when regarded as separate

whole entities, show mean angular deviations approaching zero (marginally positive) when relative curvature, using both positive and negative values according to change in flow direction (derivation explained in detail in figure 3.14), is considered. At the sampling interval used, (Δx = one tenth mile) non-zero values indicate a preferred direction of deviation from flow path, and so may represent either a low frequency meandering tendency in a much larger series, or a definite flow trend.

Indication of a flow trend would suggest the presence of a large scale process operating somehow within the landscape to cause flow elements to prefer one deviation direction over others. Since only local or relative curvature was derived, this implies that a non-random or deterministic element acting within or somehow underlying the fluvial processes is pulling flow in a preferred direction. A very large sample of deviation angles would be necessary in order to eliminate low frequency meander possibilities as the genesis of an apparent trend in the data.

Detrending data before analysis effectively removes the influence of such a trend on statistics obtained in the analysis. It is interesting, however, to note and describe trends (apart from during spectral analysis) as these might help in the definition of underlying physical processes active in the establishment of the series obtained for analysis.

b) Beaver River

The Beaver River shows the most pronounced deviation preference among the rivers studied. The series for the reach concerned shows a mean deviation of 2.7 degrees to the right. Table 4.03 shows that both

the straight reach and the upstream meanders support this trend, while the downstream section of meanders exhibits a near-zero mean deviation. The total reach of river included in the study was rather small (only 18.5 miles of flow path), and it has been mentioned that the meanders recorded in the study are misfit in a large valley that meanders at a very low frequency relative to the oscillations in the Beaver's planform geometry. The study reach constitutes a small portion of a large concave-downward bend in the main valley, accounting for the apparent right trend in the series.

Meanders on the Beaver River are marginally tighter in the upstream portion of the reach. Maps show that upstream from the study reach, the meanders tend to assume longer wavelengths relative to their amplitudes, while becoming more sharply curved in the downstream direction, shown by an increase in the mean angular deviation.

In laboratory simulations of the initiation and development of river meanders, meanders begin to form at the furthest point downstream and the sinuous pattern moves slowly in the upstream direction (Friedkin, 1945). Many scroll bars along the meanders, as shown in Figure 3.06, give evidence of the history of downstream movement of meanders. These have not been dated though, and it is not possible to interpret the rate of meander development from available data. Dating of scroll bars and data on much larger portions of the Beaver would be of assistance in the determination of patterns and processes operating on the Beaver's planform.

c) The Kootenay River:

Planform data for the Kootenay River suggest an overall tendency

toward equilibrium of angular changes in planimetric geometry. The slightly positive means derived could indicate a marginal preference for left turns (southwest flow deviation), following the geology of the region.

Bends tend to become sharper in the downstream direction, and oscillation appears most intense in the lower reaches studied, as evidenced in increasing variance.

d) The South Nahanni River:

The South Nahanni River also shows a tendency to balance reversals of curvature, as its mean deviation is also near zero. Angular statistics in Table 4.03 suggest the possibility of a slight preference for right turns, or north-west flow. This reflects the general orientation of the landscape in macro-scale.

Bends tend to become tighter in the downstream direction in all channel types of the South Nahanni series.

e) Theoretical Implications

The near-zero deviations found for the South Nahanni and Kootenay reaches suggest a tendency for rivers to keep angular turns in directional equilibrium, implying the ability of these rivers to balance chance or determined initial deviations somewhere later in the series through the establishment of counter-deviations in the flow-path. Both the Kootenay and South Nahanni series are relatively large samples of their respective rivers. The Beaver series is, in contrast, a smaller sample of its river, and it exhibits a trend not found in the other series. A preferred deviation direction is expressed in the sampled planform, but superimposition of the series on a low frequency oscillation explains the tendency.

D. INFORMATION CONTAINED IN THE SPECTRAL AND CROSS-SPECTRAL STATISTICS

Although the analysis of the depth configuration and planform of rivers, based on the first two moments of each of the respective series, allows some insight into the meandering process, the insight provided is rather limited and is little more than descriptive. Processes are identified, when at all, only in very general terms. Definition of relationship of variables is crude. Sophistication of analysis should provide more valuable information and more detailed insight.

Spectral and cross-spectral analysis are described in greater detail in Chapter 2. Appendix 2 details calculations involved in the computer program used in the derivation of the statistics used. The standard "Biomedical Computer Program" BMD:02T, expanded to take up to 3000 cases, was used in all spectral and cross-spectral computation. All calculations were conducted using a 10 % m to n ratio.

The creation of a third depth series was necessary in order for the cross-spectral analyses to be performed. The corrected series were sampled at intervals of one-tenth mile along the river path. In order to minimize error of estimate by this method, at each tenth mile, three points (two in Series B) were averaged to obtain each of the depth values used to construct the new depth series. Each new series then contained exactly as many points as its corresponding meander plan series. This is necessary in cross-spectral analysis. A possible alternative to this method, which would have maximized resolution in the depth series during the sampling process, would be interpolation of the meander series, along some best fit curve between measured points. Resolution with $\Delta x = 0.1$ miles, as in this study, is limited to processes manifesting wavelengths

of greater than 0.2 miles only. Smaller detail cannot be examined here. Interpolation of meander data would allow retention of detail in the depth series and would then provide more accurate analysis of the correspondence between meander plan and depth of flow. The sampling interval allowed adequate resolution for the purposes of this thesis, and so finer sampling and interpolation possibilities have not yet been explored. In any work requiring greater resolution, finer sampling or interpolation would have to be used. The sampling interval used here will obviously not pick out high frequency undulations such as dunes, but it is adequate in the description of large-scale processes within the river.

The use of finer sampling or interpolation raises another problem, however. Most computer programs available for spectral and cross-spectral analyses will take only a specified number of cases. In order to use the techniques effectively, a suitable combination of degree of resolution and adequate length of record must be achieved. In most instances, this will likely require the expansion of the computer programs used.

1. General

The resolution chosen for this study limits the phenomena that can actually be examined to the large-scale processes characteristic of fluvial behavior. General patterns of bed configuration fluctuations can be analyzed and cross-examined with planform geometry, but specific small patterns are bypassed. Dunes cannot be examined or described using the new series data, but the identification of pool and riffle sequences will be within the scope of the analyses. Resolution of

wavelength is limited to those greater than 0.2 miles for Δx equal to 0.1 mi.

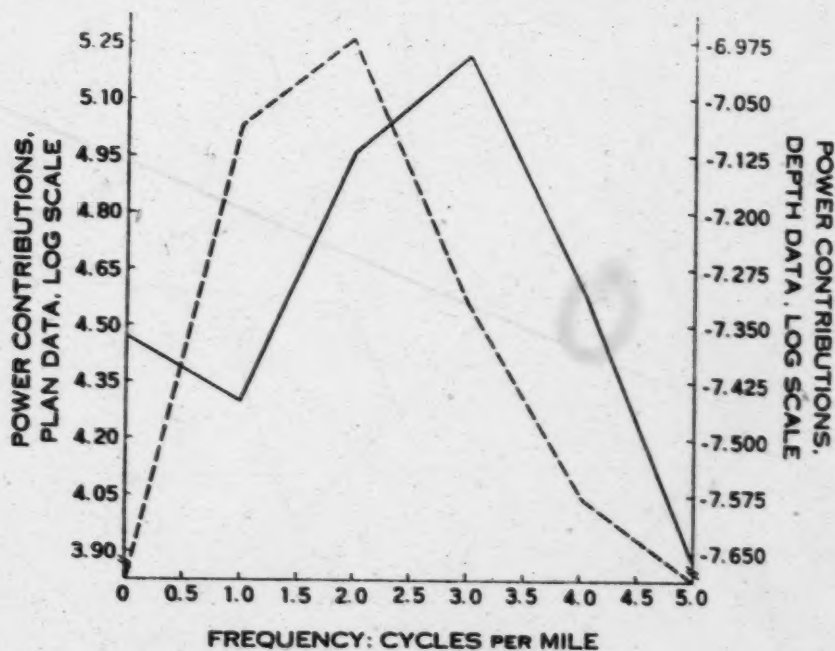
Coherence between depth and curvature spectra are generally quite low. The use of absolute curvature rather than relative curvature seems to make little difference in the coherency estimates. Resolution problems may thus be present in the coherency analysis.

2. The Beaver River

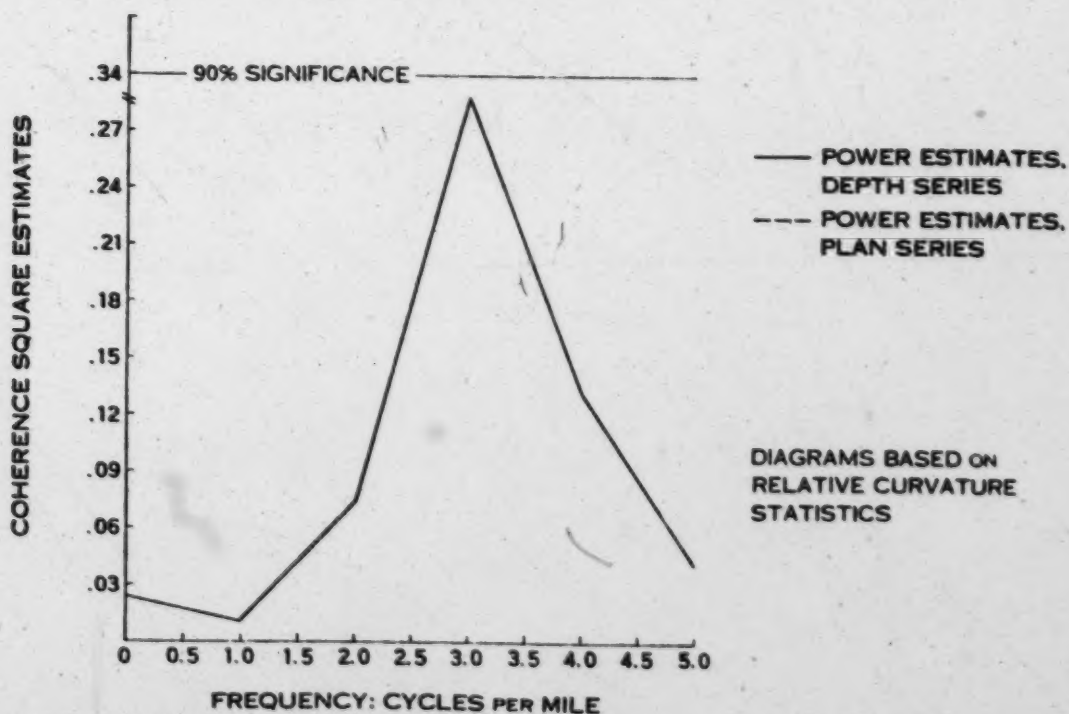
a) Depth Spectra

Figures 4.19, 4.20, 4.21, and 4.22 present spectral results for the Beaver River. Depth spectra for the Beaver River exhibit stable patterns characteristic of all individual reaches. In all cases, predominant frequencies were in the intermediate range, with wavelengths from $\frac{1}{4}$ to $\frac{1}{2}$ mile contributing the majority of power to the spectra. A tendency for shorter wavelengths, or oscillations in depth of higher frequency, to become more important in the straighter section was observed. Higher frequencies also tended to contribute a greater proportion of variance in the downstream section of meanders, indicating the maintenance of the predominant mode of wavelength in depth series with an additional increased importance of oscillations of shorter wavelength. No obvious trend of wavelength increasing downstream emerged in the depth spectra, but the sample was small, and the flow was confined within a much larger, older valley. Figures 4.19, 4.20, and 4.21 allow comparison of the three reaches in their depth spectra. Since the natural series are actually continuous, and records used in the calculation of spectral statistics are discrete, with a sampling interval of 0.1 miles,

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

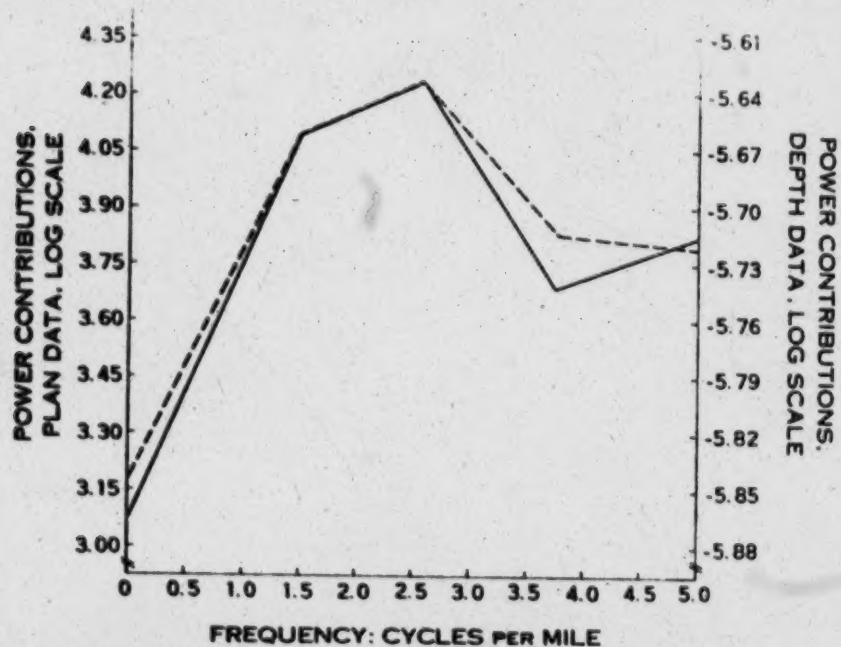


B) COHERENCE SQUARE ESTIMATES

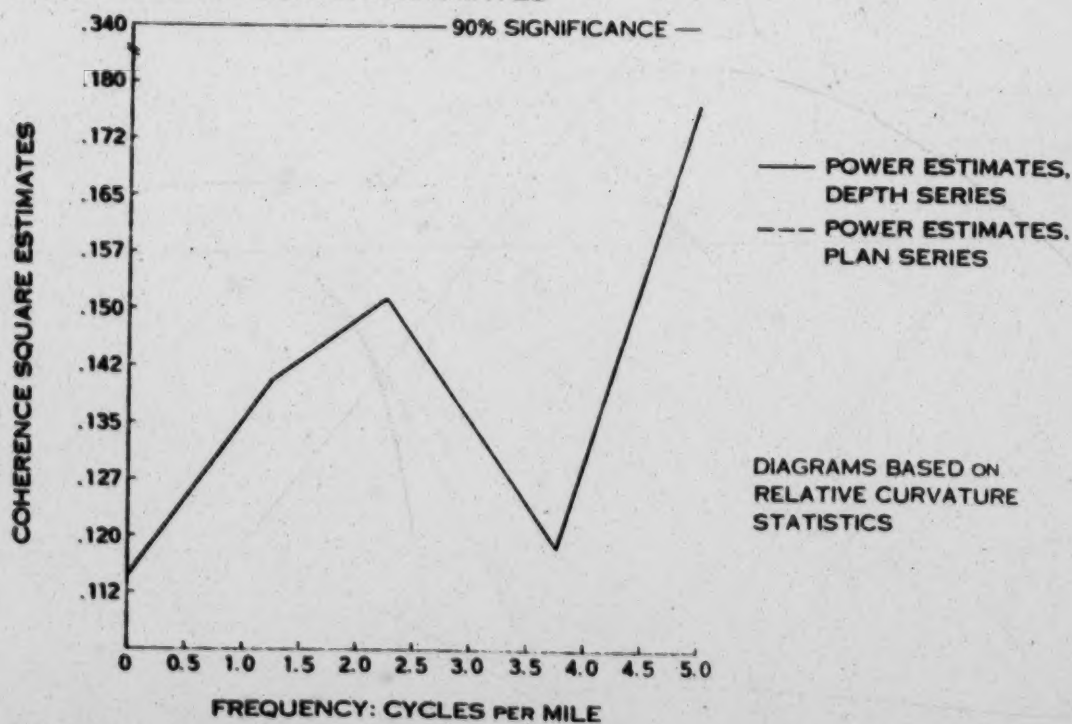


PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR UPSTREAM MEANDERS — BEAVER RIVER
FIGURE 4.19

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

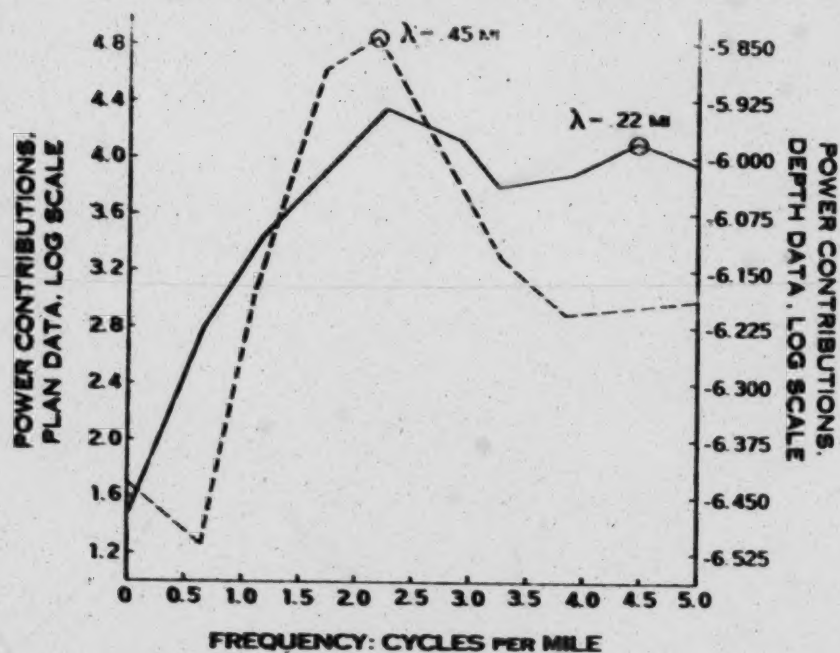


B) COHERENCE SQUARE ESTIMATES

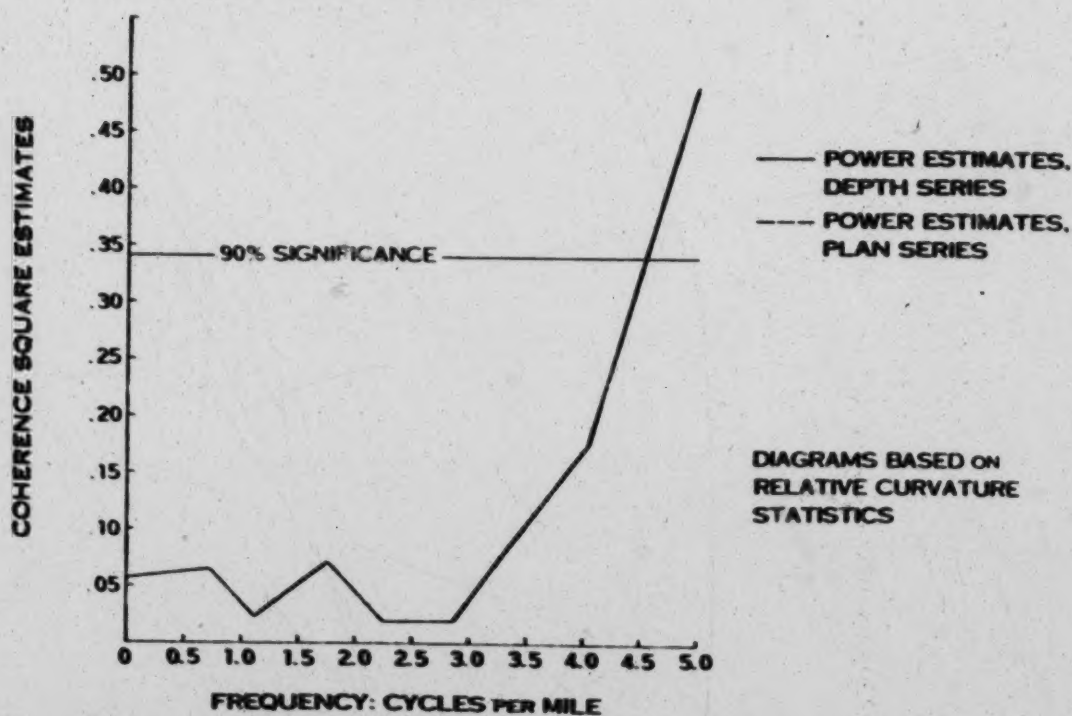


PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR STRAIGHT REACH — BEAVER RIVER
FIGURE 4.20

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

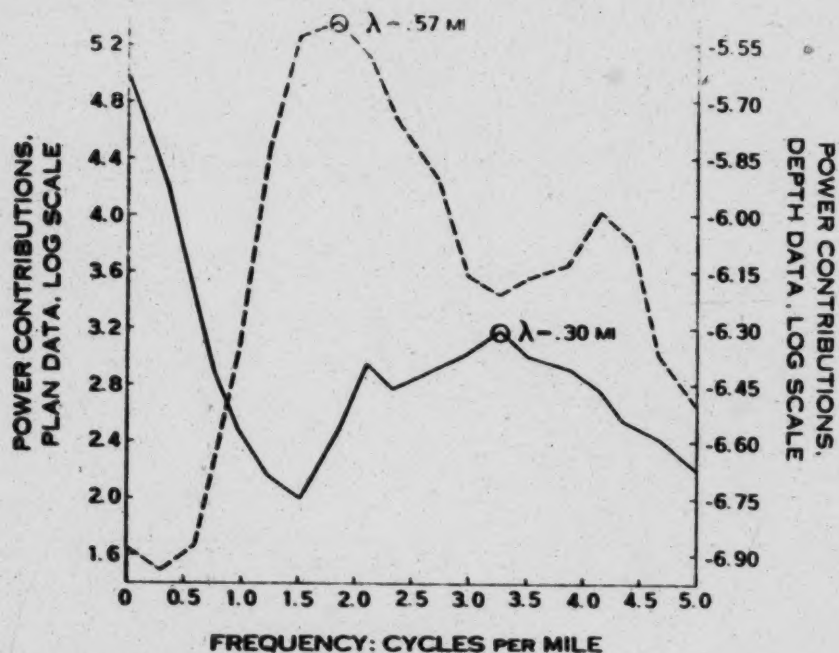


B) COHERENCE SQUARE ESTIMATES

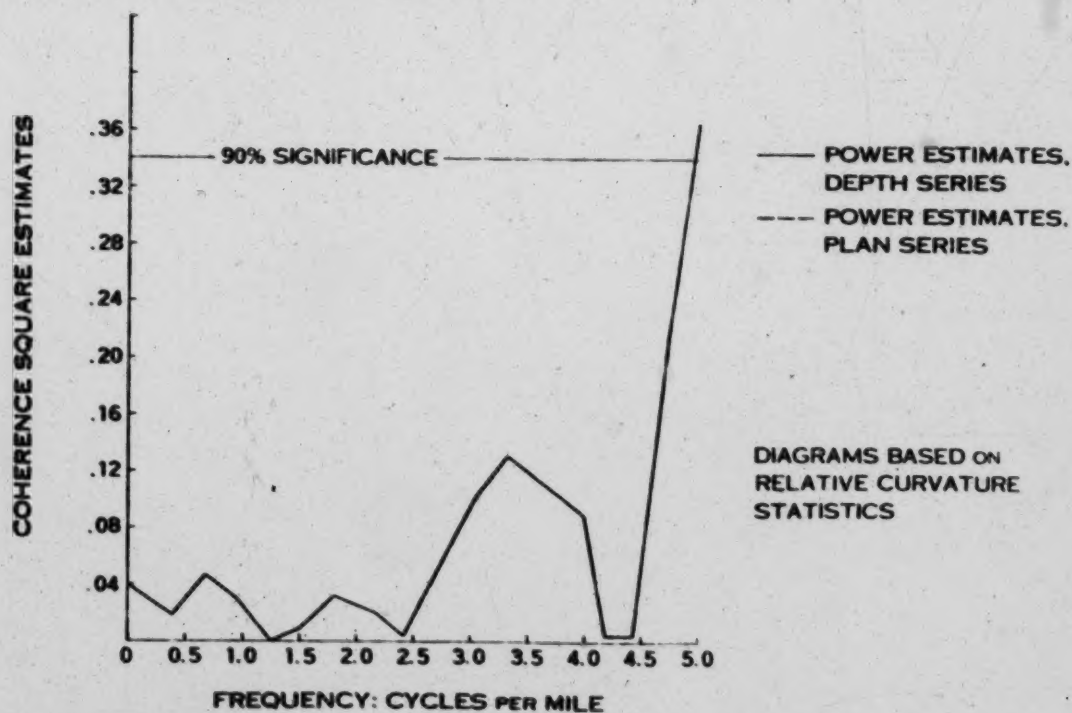


PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR
DOWNSTREAM MEANDERS — BEAVER RIVER
FIGURE 4.21

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE



B) COHERENCE SQUARE ESTIMATES



PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR ENTIRE BEAVER RIVER REACH
FIGURE 4.22

peaks and all frequencies listed are band limited.

Spectra of individual reaches resemble those expected for second-order autoregressive processes, showing a peak at intermediate frequencies. In contrast, the depth spectra for the entire series (Figure 4.22) appears slightly more as a first-order autoregressive process, with power concentrated in lower frequencies. The longer reach demonstrates the contribution of much longer wavelengths (those approaching infinite wavelength are shown as the most powerful) reflecting the influence of the older valley.

The straight section shows greater contribution of power by higher frequencies, suggesting oscillations of very short wavelength. Dunes were present in the river at the time of the survey, but resolution of the data does not allow detail to enter this larger pattern.

All reaches showed a tendency to favor depth wavelengths of 10 to 15 river widths, in conformity with the findings of Leopold, Wolman, and Miller (1964). These represent oscillations of about 3π and 5π river widths, well within the range suggested by Yalin (1971) and approximating the distribution of regions of bed erosion, as Yalin (1972) suggested.

Major wavelengths demonstrated a tendency to approximate a distance equal in magnitude to a value $\pi\bar{A}$ for the reach, where $\bar{A}$ is the average cross-sectional plane parameter (assuming a rectangular channel). $\bar{A}$ is the product of average recorded depth and mean width of the reach considered. Table 4.06 illustrates the property found.

b) Planform Spectra

The spectra of angular deviations showed a similar pattern

Table 4.06: CORRESPONDENCE OF $\pi\bar{\lambda}$ TO SPECTRAL WAVELENGTHS, BEAVER RIVER

	mean depth, ft.	mean width, ft.	depth spectral λ 's	$\pm\phi$ spectral λ 's	Coh ² λ 's (with $\pm\phi$)	$ \phi $ spectral λ 's	Coh ² λ 's with $ \phi $
entire river	3.77	180	.87 $\bar{\lambda}$ 1.27 $\bar{\lambda}$	.77 $\bar{\lambda}$ 1.47 $\bar{\lambda}$	.87 $\bar{\lambda}$ 1.47 $\bar{\lambda}$	.97 $\bar{\lambda}$.67 $\bar{\lambda}$	1.27 $\bar{\lambda}$.67 $\bar{\lambda}$.77 $\bar{\lambda}$
downstream meanders	4.27	158	1.17 $\bar{\lambda}$	1.57 $\bar{\lambda}$	.67 $\bar{\lambda}$	.67 $\bar{\lambda}$	.97 $\bar{\lambda}$
straight section	2.44	211	1.37 $\bar{\lambda}$	1.37 $\bar{\lambda}$ 2.67 $\bar{\lambda}$	1.57 $\bar{\lambda}$ 1.37 $\bar{\lambda}$	.97 $\bar{\lambda}$ 1.37 $\bar{\lambda}$	1.57 $\bar{\lambda}$ 1.37 $\bar{\lambda}$ 2.67 $\bar{\lambda}$
upstream meanders	3.96	164	.97 $\bar{\lambda}$	1.37 $\bar{\lambda}$	.97 $\bar{\lambda}$	.97 $\bar{\lambda}$.77 $\bar{\lambda}$	2.67 $\bar{\lambda}$ 1.37 $\bar{\lambda}$

for meandering sections. Variance and power patterns are more pronounced for relative than for absolute curvature, and absolute curvature tends to produce wavelengths twice those for relative curvature, as expected. Absolute curvature has a smoothing effect on the information contained in the planform series, and so shows a pattern of power more evenly distributed across frequencies.

Again, spectra tended to resemble that for the second order autoregressive process, with predominant peaks at intermediate frequencies. The plan spectra for the entire reach reflected this pattern. Figures 4.19, 4.20, 4.21, and 4.22 also give the plan spectra for the series.

Wavelengths of the meander data show a tendency to occur both at approximately twice the downstream distance of the depth wavelengths of their respective reaches, and at wavelengths equal to the wavelength of that of their corresponding depths. Figures 4.19, and 4.21 illustrate this property for both the meandering sections of the Beaver River.

Again, wavelengths shown as predominant by the spectra tend to approximate values of $\pi\bar{A}$ for their reaches (refer to Table 4.06). Plan wavelengths range between 6 and 20 river widths in distance, or, between about 2π and 6π widths, as suggested by Yalin (1971).

c) Coherency Spectra

Spectra of coherence-squared shows that the best relationship between depth and plan series occur at wavelengths equal to those found to be dominant for the depth series, as well as at wavelengths twice the value favored by the depth. The relationship is most striking in the straight section, as shown in Figure 4.20, although these coherences

are not statistically significant.

Most coherence-squared estimates are fairly low, however (less than .34, critical value for 90% tolerance computed according to Panofsky and Brier, 1963), and only the extremely high frequencies with relative curvature showed a significant wavelength. These are of the order of maximum possible resolution, or 0.2 miles. This contradicts the findings of Chang (1969) concerning absolute curvature, but supports direct observational evidence that plan and depth changes correlate spatially at high frequencies. Figures 4.19, 4.20, 4.21 and 4.22 show spectra and coherency using relative curvature for the same reaches using absolute curvature. Since, in general, absolute curvature does not appear to improve coherency estimates with respect to significance, and since direction of flow does seem to matter to the river in its activity (tendency to equalize reversals of curvature), relative curvature data is the basis of most coherency observations for this study.

Coherence-squared wavelengths show the relationship to their mean rectangular cross-sectional areas. These dominant wavelengths are equivalent to 10 and 20 river widths, and 7 river widths, ranging between 2 and 20 river widths.

d) Low Frequency Waves

Predominance of any low frequency activity seems de-emphasized by this analysis of specific reaches. Importance of power concentrations at long wavelengths is more apparent in the spectra derived for both plan and depth series of the entire reach studied. Waves ranging from infinite length to 1.7 miles in length are given more (power) in these

spectra. Power concentrations decrease toward intermediate frequencies and resume important contributions only at higher frequencies (1.471 cycles per mile and greater) or shorter wavelengths (0.7 miles). The higher frequency fluctuations of the Beaver River are evident in the smaller reaches, but the large scale activity is not obvious without the inclusion of much more data. Low frequency waves become more precisely defined with increase in sample size. (See figures 4.19-4.22.)

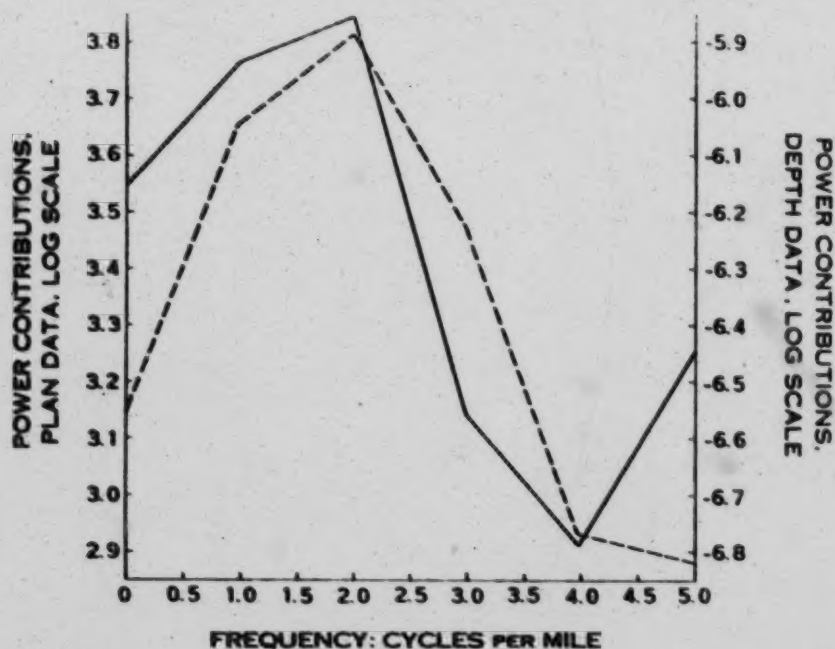
3. Kootenay River

Figures 4.23, 4.24, and 4.26 show spectral results for the Kootenay River. A larger sample of points was obtained for the Kootenay than for the Beaver River. The Kootenay sample also included a larger variety of channel types and exhibited much less regular meandering than did the Beaver River. Both planform and depth series show a more even distribution of power among frequencies, emphasizing the presence of a number of wavelengths, none of which sufficiently summarizes the nature of the Kootenay series. Consideration of many wavelengths is necessary to gain an understanding of processes involved in the formation of the patterns derived. Table 4.07 summarizes the spectral findings for the Kootenay.

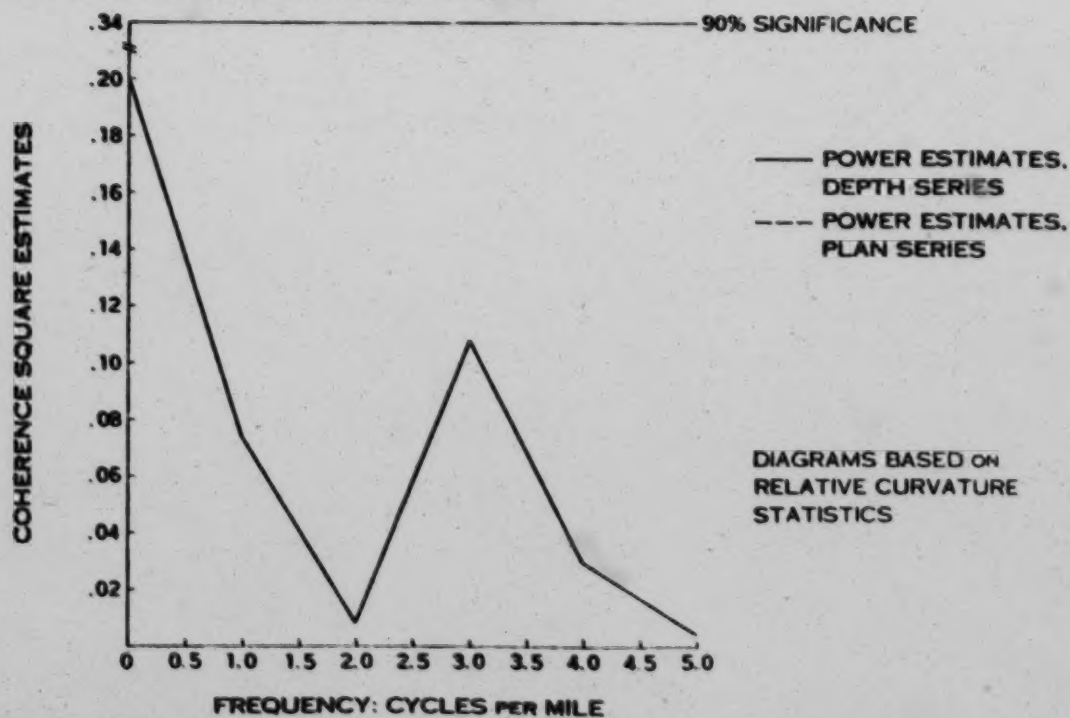
a) Depth Spectra

Depth spectra for the Kootenay show an increase in wavelength downstream along the river. This pattern is less obvious from maps but shows up clearly in the spectra of the bed configuration. Figures 4.23, 4.24, 4.25, and 4.26 show this tendency. Concentrations in power are clear at several frequencies. Lowest frequencies contain the majority of spectral power evidenced in the series, and power increases,

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

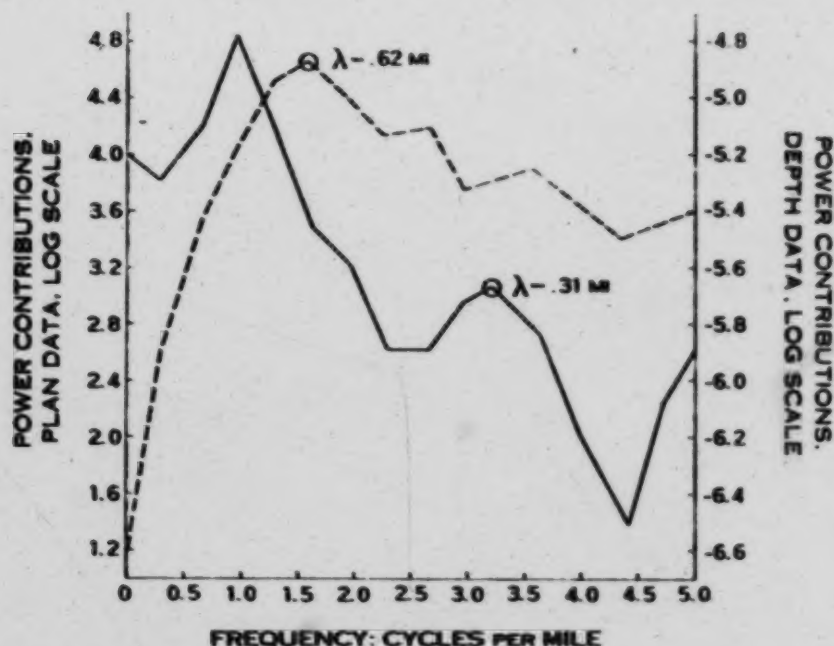


B) COHERENCE SQUARE ESTIMATES

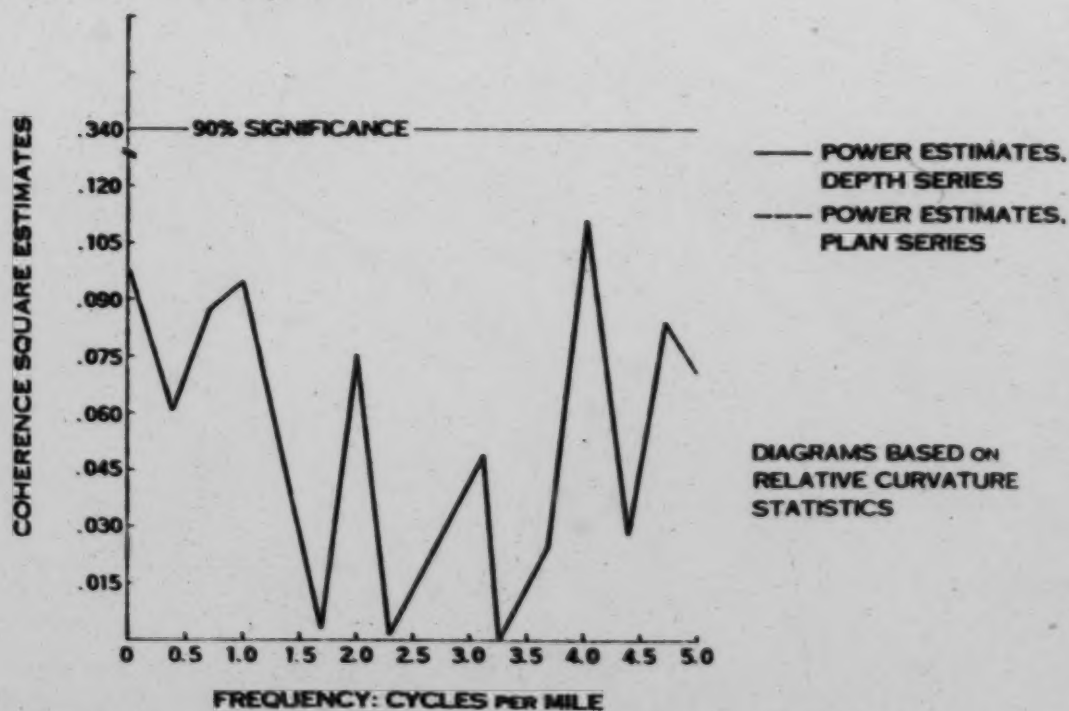


PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR THE KOOTENAY RIVER BRAIDED SECTION
FIGURE 4.23

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE



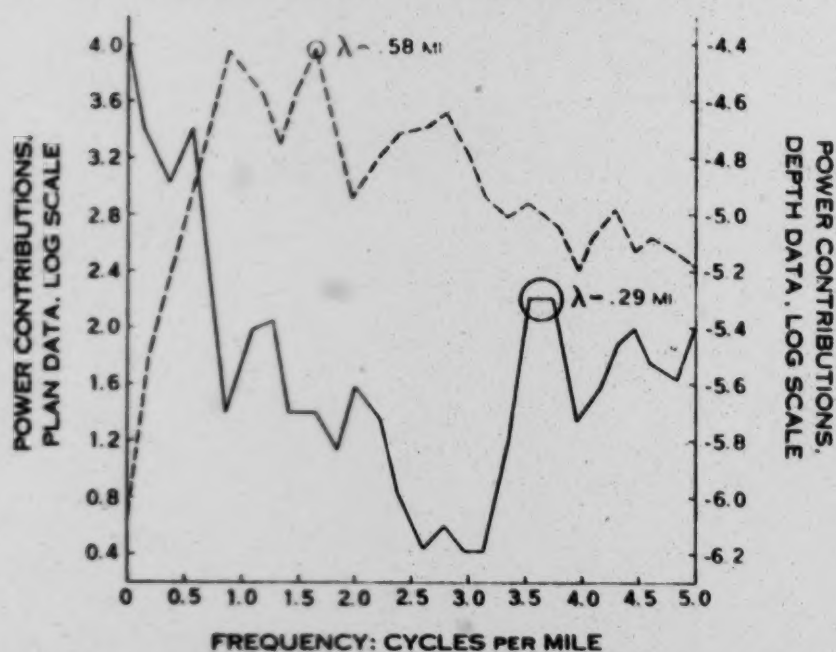
B) COHERENCE SQUARE ESTIMATES



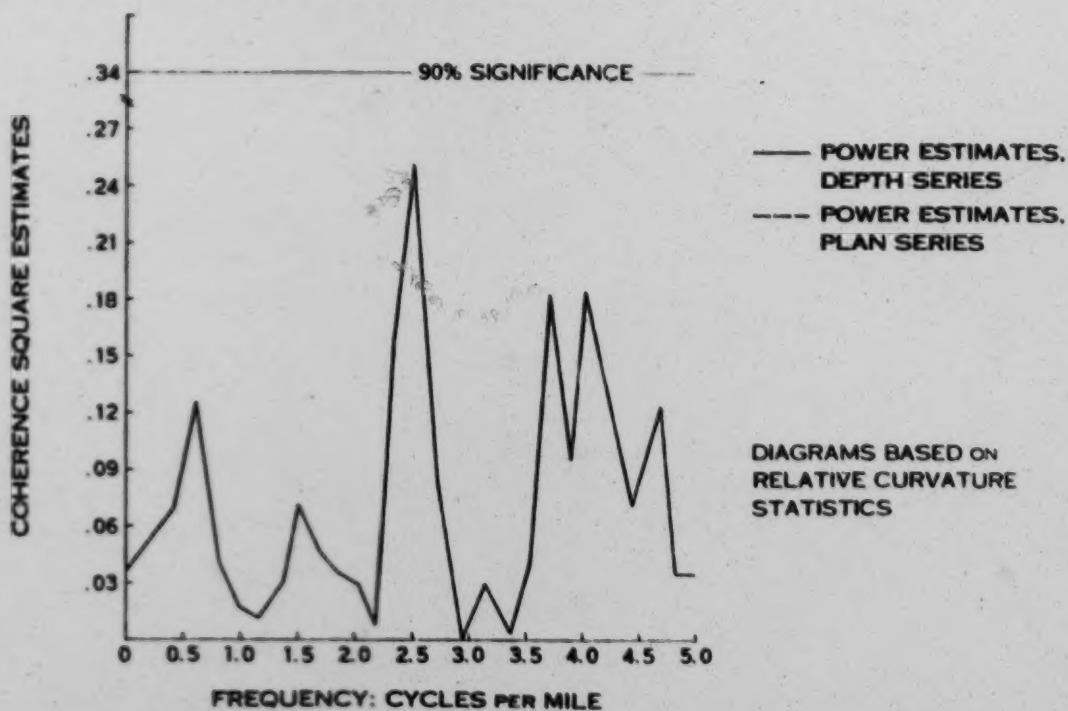
PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR DOWNSTREAM SEMI-ENTRENCHED SECTION, KOOTENAY RIVER

FIGURE 4.24

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

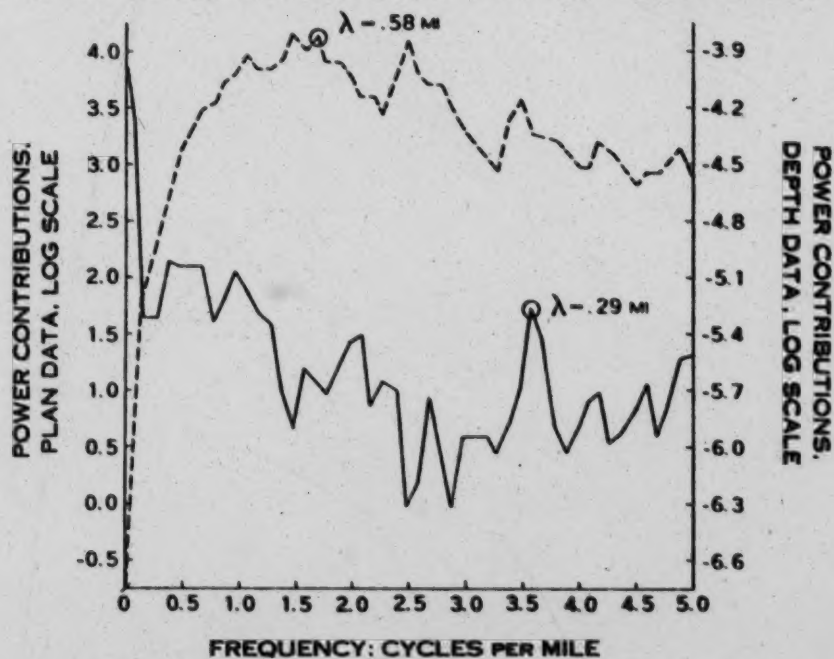


B) COHERENCE SQUARE ESTIMATES

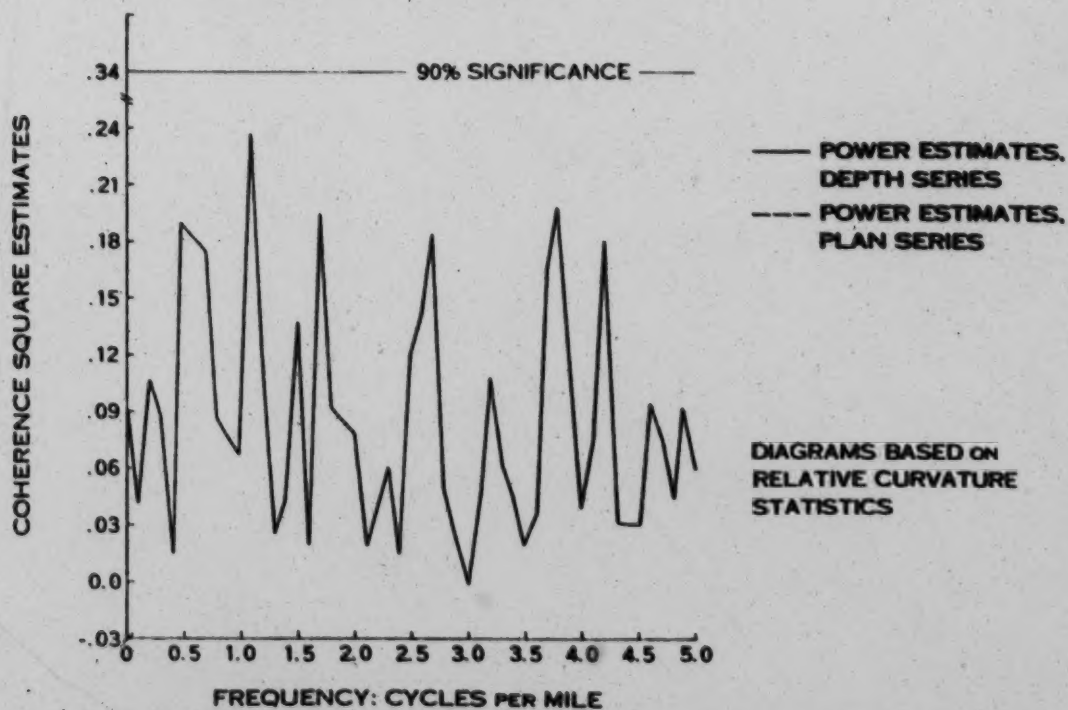


PLAN AND DEPTH SPECTRA A) AND COHERENCE
SQUARES B) FOR
UPSTREAM IRREGULAR MEANDERS, KOOTENAY RIVER
FIGURE 4.25

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE



B) COHERENCE SQUARE ESTIMATES



PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR ENTIRE KOOTENAY REACH SURVEYED
FIGURE 4.26

though less rapidly, towards very high frequencies. Three scales of activity between lowest and highest frequencies seem important. Wavelengths of 0.9 to 0.7 miles, 0.5 miles, and about 0.3 miles stand out as intermediate spectral peaks in all reaches considered, among which the wavelength 0.3 miles seems most "powerful". These define the irregular meandering behavior of the Kootenay.

Predominant wavelengths occur at between about 5 and 20 river widths, although some longer wavelengths are found with less power. Wavelengths for entrenched sections are apparently longer than those in less confined sections. Table 4.07 lists major spectral statistics for the Kootenay River.

A preference for coherence wavelengths to occur as multiples of πw is evident in the table. Values between πw and $8 \pi w$ are favored in all but a preference towards one of odd or even number multiples is not obvious.

All depth series appear to assume the form of a second-order autoregressive process. Peaks at intermediate frequencies interrupt the curve. All reaches exhibit this pattern, which is also consistent with observations on the Beaver's spectra.

b) Planform Spectra

Spectra of angular deviations also show increasing wavelength downstream for the Kootenay. Again, several intermediate scales of activity contain the power of the series. Predominant wavelengths occur as 1.0 and .5, .6 and .3, and .2 miles, with .6 miles concentrating more power than any other wavelength. This compares favorably with spectral power indicated by the Beaver, as planform wavelengths are twice depth wavelengths for the Kootenay as well. The pool-and-riffle

Table 4.07: GENERALIZED SPECTRAL STATISTICS FOR THE KOOTENAY RIVER

	Depths			Plan ($\pm\phi$)			Coh ²		
	λ , mi	$\frac{\lambda}{w}$	$\frac{\lambda}{\pi w}$	λ , mi	$\frac{\lambda}{w}$	$\frac{\lambda}{\pi w}$	λ , mi	$\frac{\lambda}{w}$	$\frac{\lambda}{\pi w}$
1. Braids	1.00	16.7	5.3	.50	8.3	2.7	.33	5.6	1.8
	.50	8.3	2.7	1.00	16.7	5.3			
	.20	3.3	1.1	.33	5.6	1.8			
2. Downstream	1.00	25.0	8.0	.60	15	4.8	.25	6.26	1.99
	.30	7.5	2.4	.21	5.4	1.7	1.00	25.0	8.0
	.21	5.4	1.7	.27	6.8	2.2	.50	12.5	3.98
3. Upstream	.77	18.1	5.8	.93	25.3	8.0	1.8	42.2	13.4
				.6	15	4.8			
	.49	11.5	3.7				.38	9.0	2.9
	.30	7.5	2.4	.36			.67	15.8	5.0
	.36	8.4	2.7		8.4	2.7	0.27	6.3	2.0
	0.27	6.3	2.0	.23	5.5	1.8	0.21	5.1	1.6
	0.28	6.7	2.1						
	.20	5.4	1.7						

Table 4.08: $\pi \bar{\lambda}$ RELATIONSHIP FOR THE KOOTENAY RIVER

	mean depth, ft.	mean width, ft.	depth spectral $\bar{\lambda}$'s	$\pm \theta$ spectral $\bar{\lambda}$'s	Coh ² $\bar{\lambda}$'s (with $\pm \theta$)	$ \theta $ spectral $\bar{\lambda}$'s	Coh ² $\bar{\lambda}$'s with $ \theta $
1. braids	7.5	317	.7 $\bar{\lambda}$	.7 $\bar{\lambda}$	.2 $\bar{\lambda}$	.2 $\bar{\lambda}$	.2 $\bar{\lambda}$
			.4 $\bar{\lambda}$	.4 $\bar{\lambda}$		1.1 $\bar{\lambda}$	.3 $\bar{\lambda}$
				.2 $\bar{\lambda}$			1.1 $\bar{\lambda}$
2. downstream	9.0	211	.9 $\bar{\lambda}$	.4 $\bar{\lambda}$	.9 $\bar{\lambda}$	1.3 $\bar{\lambda}$	.3 $\bar{\lambda}$
irregular			.3 $\bar{\lambda}$	.2 $\bar{\lambda}$	.3 $\bar{\lambda}$	.4 $\bar{\lambda}$	.5 $\bar{\lambda}$
meanders						.3 $\bar{\lambda}$	1.3 $\bar{\lambda}$
3. upstream	8.5	225	.7 $\bar{\lambda}$	1.0 $\bar{\lambda}$	1.6 $\bar{\lambda}$	.3 $\bar{\lambda}$	.3 $\bar{\lambda}$
irregular			.4 $\bar{\lambda}$	.6 $\bar{\lambda}$	.4 $\bar{\lambda}$	.5 $\bar{\lambda}$	.9 $\bar{\lambda}$
meanders				.3 $\bar{\lambda}$	.6 $\bar{\lambda}$	1.2 $\bar{\lambda}$	.6 $\bar{\lambda}$
				.2 $\bar{\lambda}$	.2 $\bar{\lambda}$		

sequence postulated by Langbein and Leopold (1966) seems to hold for this semi-entrenched river, including reaches that vary from braided to canyoned, as well as for the regularly meandering sand-bed Beaver. Figures 4.24, 4.25, and 4.26 demonstrate this characteristic. Table 4.08 presents the $\pi\bar{\lambda}$ -wavelength relationship for the Kootenay River. The relationship is not as strong for this river, but it is still noticeable. Planform wavelengths occur as 5.4 to 16 river widths. Figures 4.23 to 4.26 give the plan spectra for the Kootenay.

c) Coherency

Table 4.07 shows a strong relationship between predominant wavelengths and πw , as wavelengths appear to vary between about 2π widths and 8π widths as multiples of π . Coherence squared estimates are best at .3 to .4 and .6 mile wavelengths, consistent with the univariate spectral suggestions for the reaches. However, none of the estimates are significant at 90%. Marginal increase in wavelength is noted for downstream reaches even in the coherency estimates.

Coherency also assumes the pattern of the second order autoregressive process. Figure 4.23 to 4.26 demonstrates the similarity. Intermediate wavelengths contribute substantially to the power of both plan and depth spectra and this is reflected in the coherency estimates.

d) Low Frequency Waves

Presence of low frequency waves, having wavelengths in the order of 2 to 3 miles (and greater) is shown to concentrate some, though less, power in the Kootenay series than in the Beaver series, for both depth and plan. The presence of low frequency activity is not emphasized in analyses of the whole study reach. These waves represent oscillations of

the order 10^2 river widths. Strong geologic controls may account for much of the low frequency activity.

4. South Nahanni River

Profiles were obtained during both the upstream and downstream navigation of the South Nahanni River. Navigators of the boats believed a thalweg survey in either direction too dangerous for records, equipment, and personnel, and so the "anti-thalweg" was obtained both for the upstream and downstream traverses. The anti-thalweg is the channel of least velocity in the river. It now seems a thalweg survey would have been possible, and plans for future work include this possibility.

Series B was surveyed as boats advanced downstream, and so it was a much faster survey, incurring loss of detail relative to series A. Navigation problems in the collection of series B also render it the least stable, and so observations are based on series A for the most part. Upstream surveys proved the least difficult throughout the study.

Table 4.04 shows that depth data differ for the upstream and downstream surveys. Change in river stage during the time period over which both surveys were conducted accounts for some change in depth, but it seems apparent that the series differences are also due to the direction of travel on the river and location of boat across the river. Two dimensional bed mapping would solve this problem, but it is costly and time consuming and was beyond the scope of this study.

Series A was chosen as the most representative sample as flow stage, boat speed, and maintenance of desired route were all most stable during the upstream traverse. Series A also contains information for reaches further upstream than included in the Series B record. The

upstream profile also allows better data resolution, and so it forms the basis of discussion for analysis of the South Nahanni River.

a) Depth Spectra

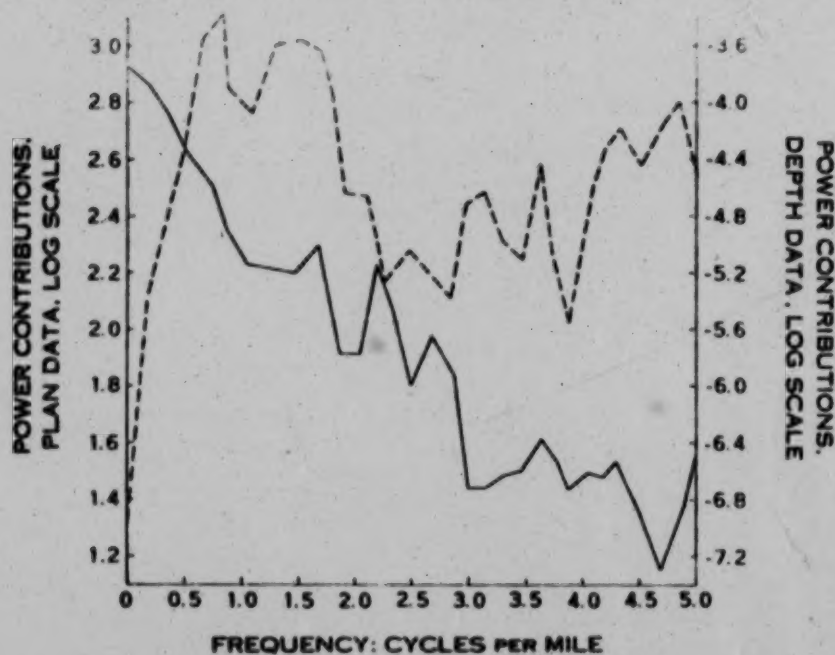
Depth spectral statistics for the South Nahanni River support the trends observed for the Beaver and Kootenay as wavelengths increase downstream along the path of flow. Comparison of the major meandering sections, First Canyon, the section including Second and Third Canyon, and Fourth Canyon, corroborate this theory even though they are all deeply incised canyons. This again seems to extend the scope of the observation. The processes involved in meandering seem to create similarities in bed configuration in spite of the variety of bed materials and bank constraints encountered. This implies that part of the meandering form can be attributed directly to streamflow processes, regardless of bed and bank materials.

The pattern established as concentration of power for the depth series follows the pattern of power concentrations in planform of twice the depth wavelength is also present in the South Nahanni data. The pattern occurs in both the canyoned and braided sections and is shown in Figures 4.27, 4.28, 4.29 and 4.30.

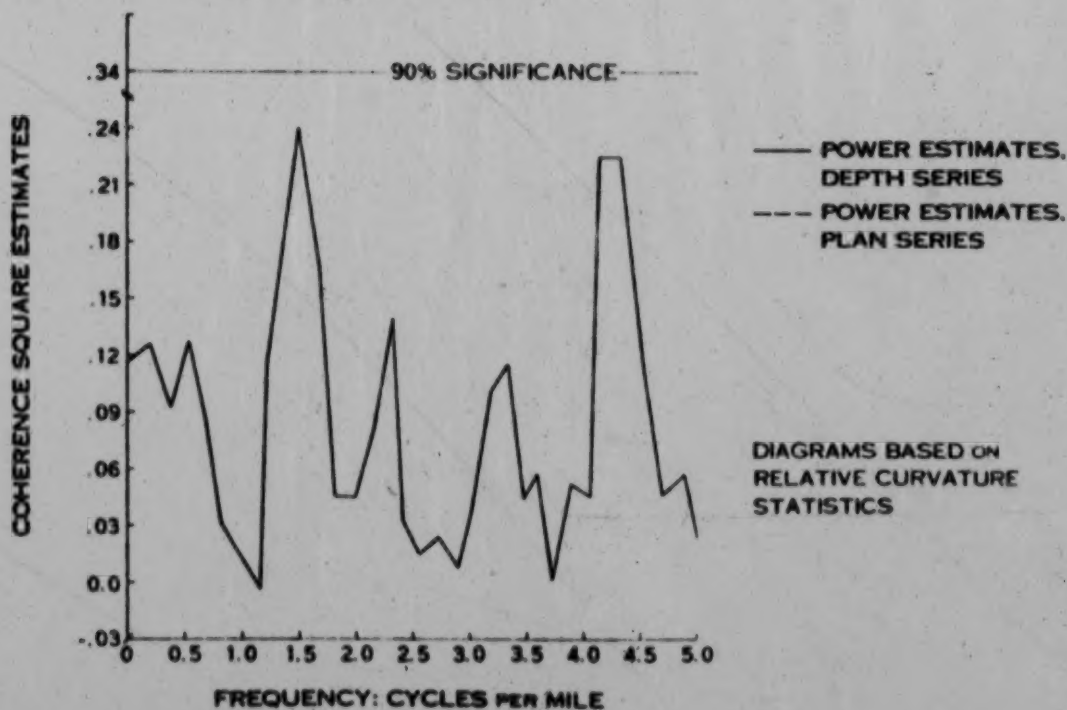
Power concentrations for the depth series occur in low frequencies and diminish as wavelengths decrease. This implies that longer wavelengths of bed undulations characterize the bed form better than shorter wavelengths, suggesting that generating processes act in the South Nahanni on a large scale. Lower frequencies contain more power for the South Nahanni than for either the Kootenay or Beaver Rivers.

Table 4.09 depicts the relationship between wavelengths and Πw

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

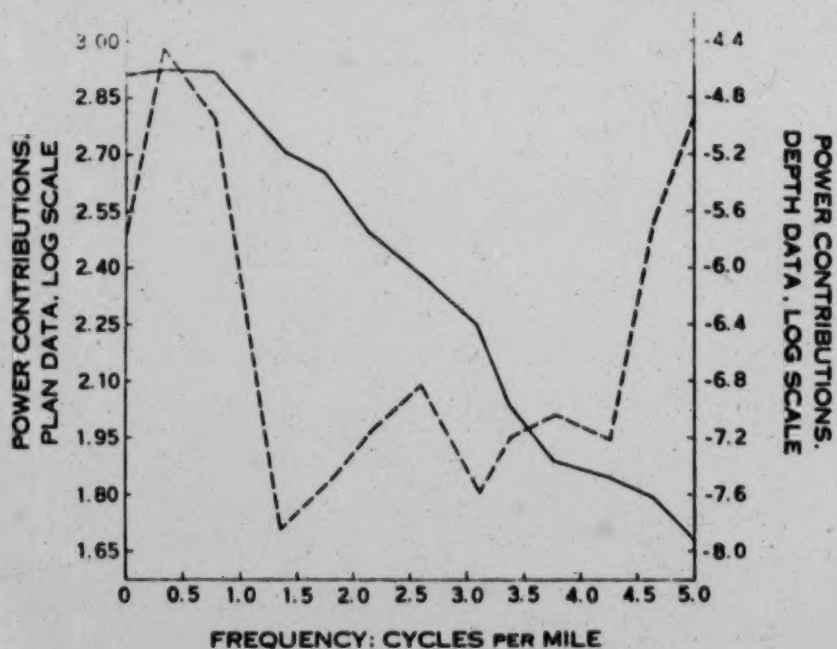


B) COHERENCE SQUARE ESTIMATES

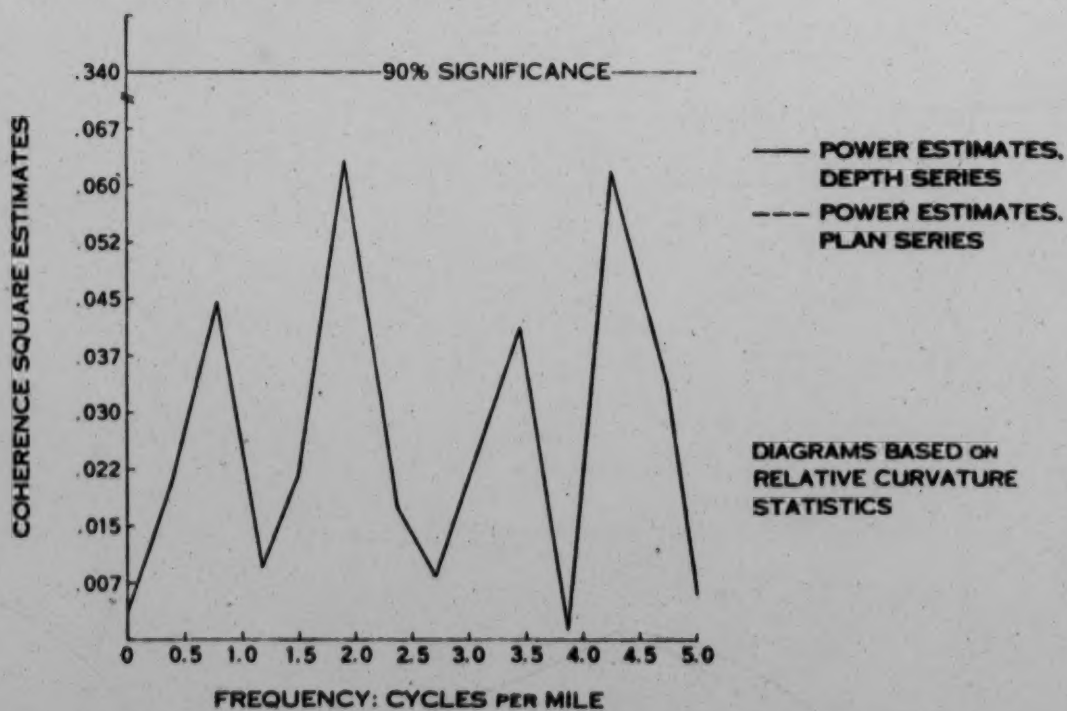


PLAN AND DEPTH SPECTRA A) AND COHERENCE
SQUARES B) FOR DOWNSTREAM BRAIDS,
SOUTH NAHANNI RIVER, MILES 81-115
FIGURE 4.27

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

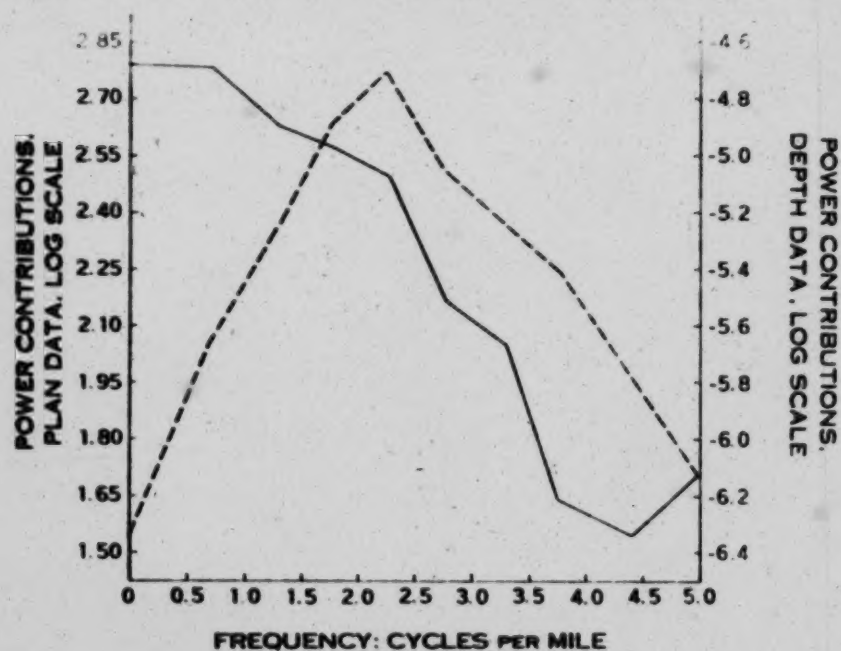


B) COHERENCE SQUARE ESTIMATES

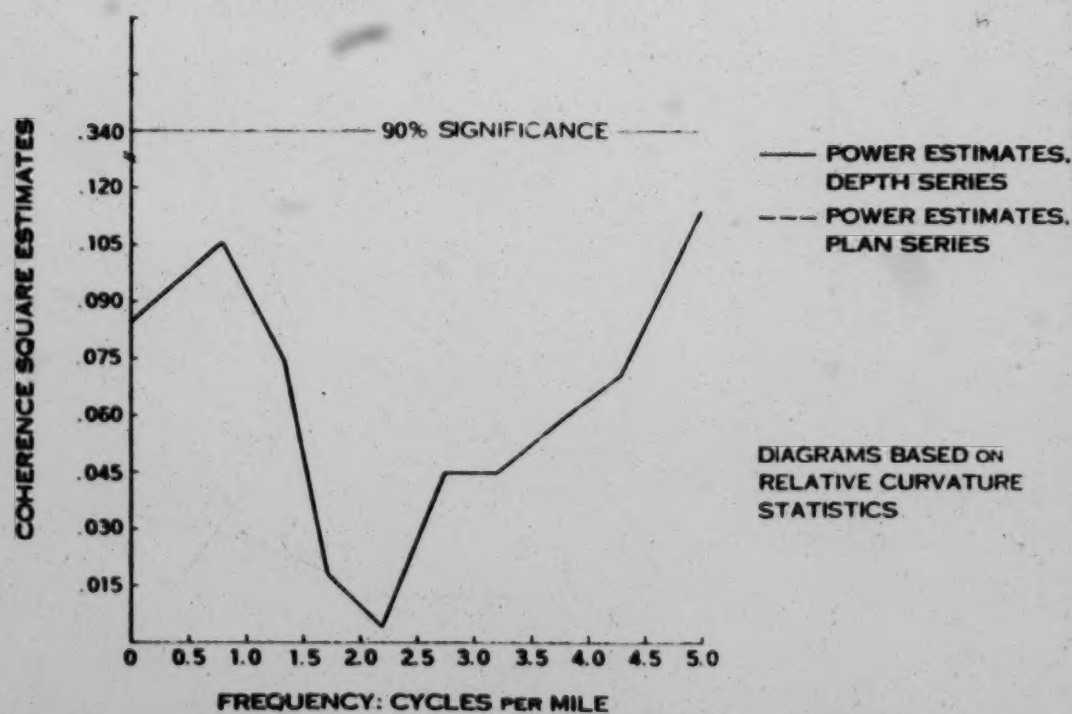


PLAN AND DEPTH SPECTRA A) AND COHERENCE
SQUARES B) FOR
FIRST CANYON, SOUTH NAHANNI RIVER
FIGURE 4.28

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE

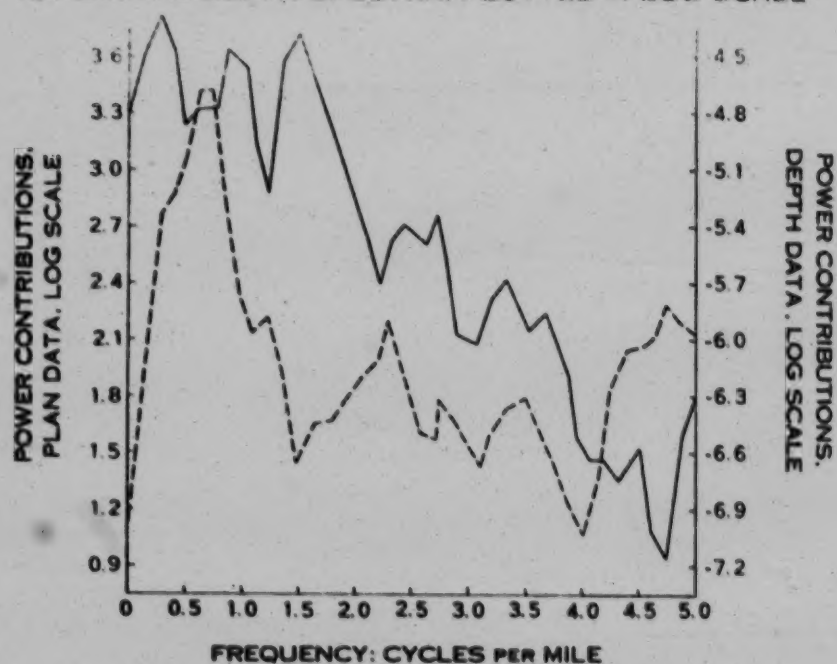


B) COHERENCE SQUARE ESTIMATES

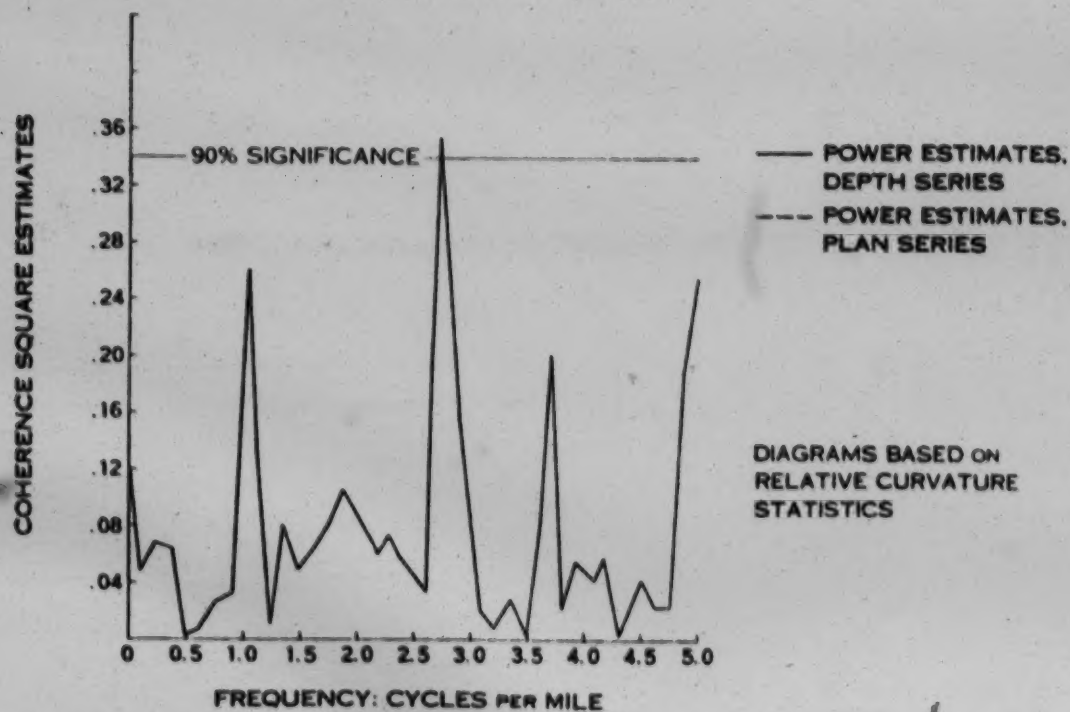


PLAN AND DEPTH SPECTRA A) AND COHERENCE SQUARES B) FOR BRAIDS BETWEEN FIRST AND SECOND CANYONS, SOUTH NAHANNI RIVER
FIGURE 4.29

A) PLAN AND DEPTH SPECTRA, PLOTTED IN LOG SCALE



B) COHERENCE SQUARE ESTIMATES



PLAN AND DEPTH SPECTRA A) AND COHERENCE
SQUARES B) FOR
SECOND AND THIRD CANYONS, SOUTH NAHANNI RIVER
FIGURE 4.30

Table 4.09: SOUTH NAHANNI SPECTRAL STATISTICS

	depth			plan $\dagger \phi$			Coh ²		
	λ , m	$\frac{\lambda}{w}$	$\frac{\lambda}{\pi w}$	λ , m	$\frac{\lambda}{w}$	$\frac{\lambda}{\pi w}$	λ , m	$\frac{\lambda}{w}$	$\frac{\lambda}{\pi w}$
1. downstream braids									
	.20	1.7	.5	1.3	11.3	3.6	.66	5.7	1.8
	.27	2.4	.75	.73	6.3	2.0	.44	3.8	1.2
	.44	3.8	1.2	.31	2.7	.9	.30	2.6	.8
	.60	5.2	1.6	.27	2.4	.75			
	3.3	28.5	9.1						
2. first canyon									
	2.59	31.3	10.0	2.59	31.3	10.0	1.3	15.7	5.0
	1.30	15.7	5.0	1.30	15.7	5.0	.52	6.3	2.0
	.52	6.3	2.0	.37	4.5	1.4	.28	3.5	1.1
				.26	3.13	1.0	.24	2.8	.9
				.02	2.4	.8			
3. second & third									
	7.8	77.3	24.6				2.6	25.7	8.2
	3.9	38.6	12.3	1.5	15.4	4.9	.98	9.7	3.1
	.65	6.4	2.0	1.3	12.9	4.1	.35	3.5	1.1
	1.1	11.0	3.5	.4	4.3	1.4	.27	2.7	.8
	.37	4.5	1.4	.2	2.1	.7	.2	2.0	.6

Table 4.09: Continued

	depth		plan ± 0		Coh ²	
	λ, m	$\frac{\lambda}{w}$	λ, m	$\frac{\lambda}{w}$	λ, m	$\frac{\lambda}{w}$
4. intermediate	1.8	15.9	.45	4.0	1.8	15.9
braids	.2	1.8	.36	3.2	.36	3.2
						5.0
						1.0

for the river. Wavelengths again tend to approximate multiples of πw between about 1π and 20 river widths, consistent with the findings of Yalin (1972).

The depth spectrum for both the Second-Third Canyons and First Canyon best resembles a second-order autoregressive process, while the spectrum for the braided sections resemble more a first-order autoregressive scheme.

b) Planform Spectra

The tendency for plan wavelength to increase downstream is apparent in the South Nahanni River. Plan seems related to bed configuration where bed undulations occur mainly at twice the frequency of plan oscillations. Table 4.09 shows the relationship found. Power at similar frequencies also shows a relationship between plan and depth.

In general, variance spectra of the South Nahanni's main reaches show power to be concentrated in both low and high frequencies. Occasional peaks in intermediate frequencies also concentrate power, and these are echoed in corresponding depth series spectra. Concentrations occur throughout the plan spectra at frequencies less than .385 cycles per mile and greater than 4.231 cycles per mile, representing wavelengths greater than 2.59 miles and less than .24 miles. Preferred intermediate frequencies include 2.308 cycles per mile ($\lambda = .43$ miles) and 3.590 to 3.718 cycles per mile ($\lambda = .28$ to .27 miles). The limit of resolution according to the sampling interval chosen is 0.1 mile, and so the analysis of wavelengths smaller than 0.2 miles was not possible from this data.

Plan spectra show wavelengths of about 2 to 40 river widths to

occur in the spectra.

Plan spectra typically appear as second-order autoregressive processes, with major power contributions within intermediate frequencies. Figures 4.27, 4.28, 4.29, and 4.30 show plan spectra for the Nahanni reaches.

c) Coherency

Low frequency coherence squares were generally high relative to those of other frequencies. Other preferred frequencies occurred as intermediate bands, and power in very high frequencies was small.

Table 4.08 shows coherency wavelengths.

A tendency for wavelengths to occur as multiples of w is evident. First Canyon, the most regularly meandering section, shows the relationship best. The coherency spectrum of First Canyon is presented in Figure 4.28.

Activity appears concentrated in wavelengths of between 2 and 20 river widths. Coherency is generally low using relative curvature data, and only a wavelength of .35 miles is significant at 90% tolerance limits. The use of absolute curvature improves the coherency somewhat, but still only three significant wavelengths, at 1.1, .37, and .36 miles are found. These represent distances of 3.4 and 1.1 river widths.

Coherency spectra are multi-peaked but tend to resemble second-order autoregressive processes.

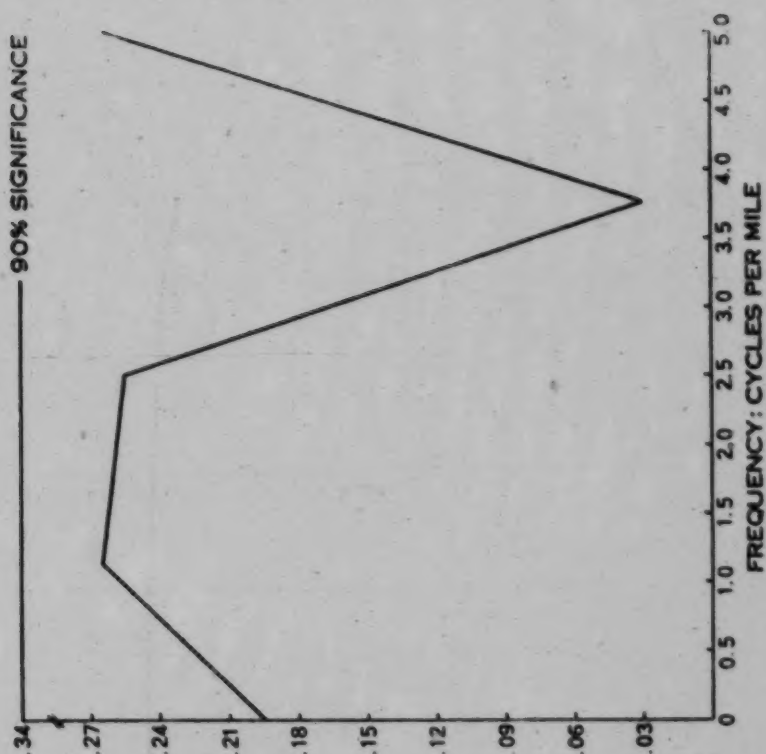
5. General

The spectra of all rivers studied generally tend to approximate second-order autoregressive processes, characterized by predominance of

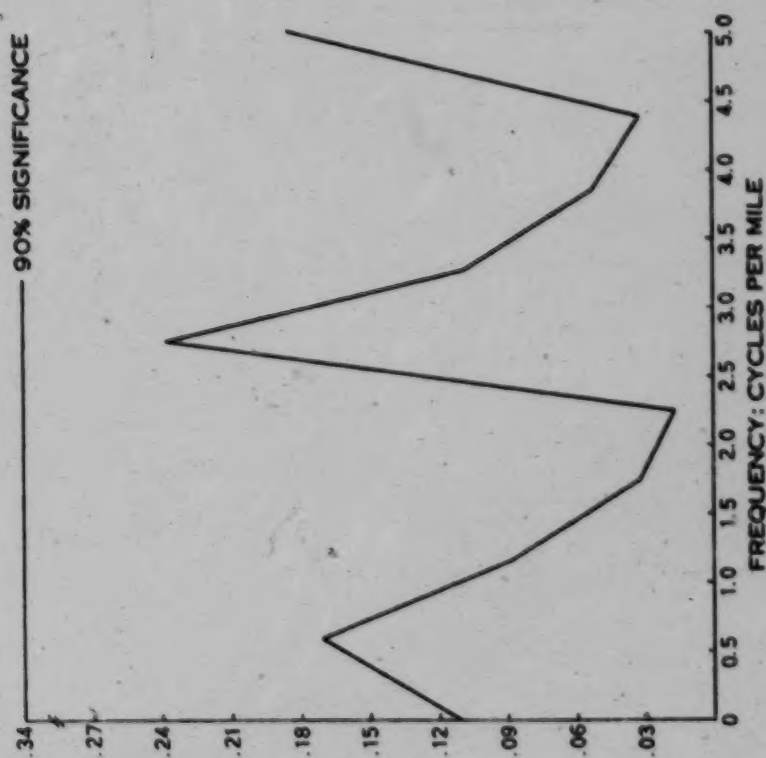
intermediate frequencies within the data. This observation corroborates the findings of Ashida and Tanaka (1967), Nordin and Algert (1966), and Hino (1968) who proposed Markov second-order linear models for the occurrence of sand waves and bed roughness.

Wavelengths generally vary between about 2 and 20 river widths, consistent with Yalin's findings (1972).

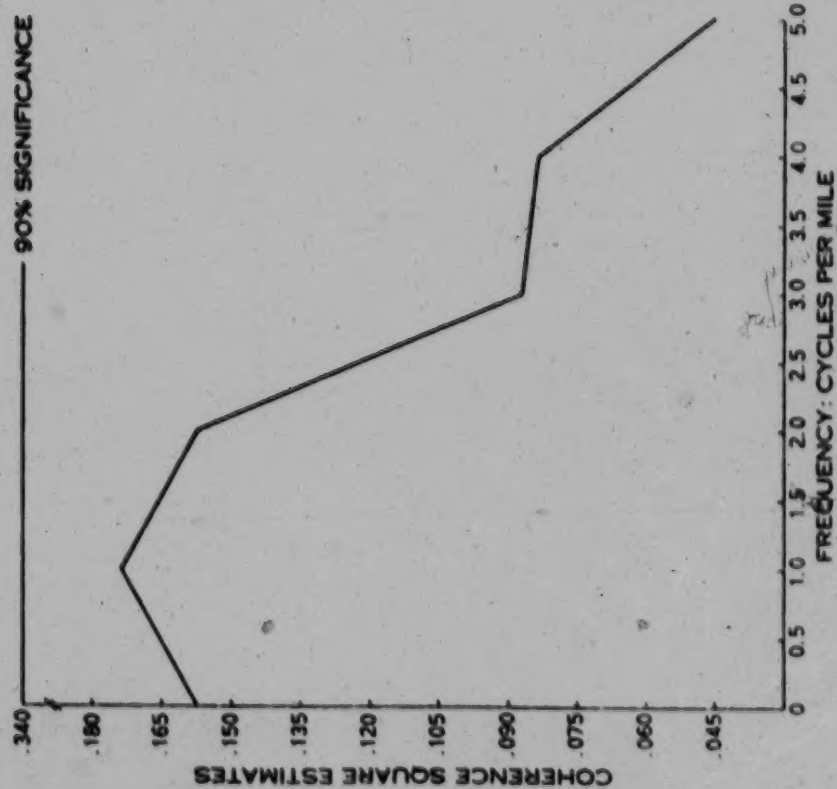
Pool-and-riffle sequences appear in the spectra for all types of reaches. The relationship, where plan wavelengths are twice depth wavelengths, is strongest in meandering and straight sections, but also occurs in braided channel.



COHERENCE SQUARE ESTIMATES, USING
| ϕ |, FOR BEAVER RIVER MEANDERS
FIGURE 4.31

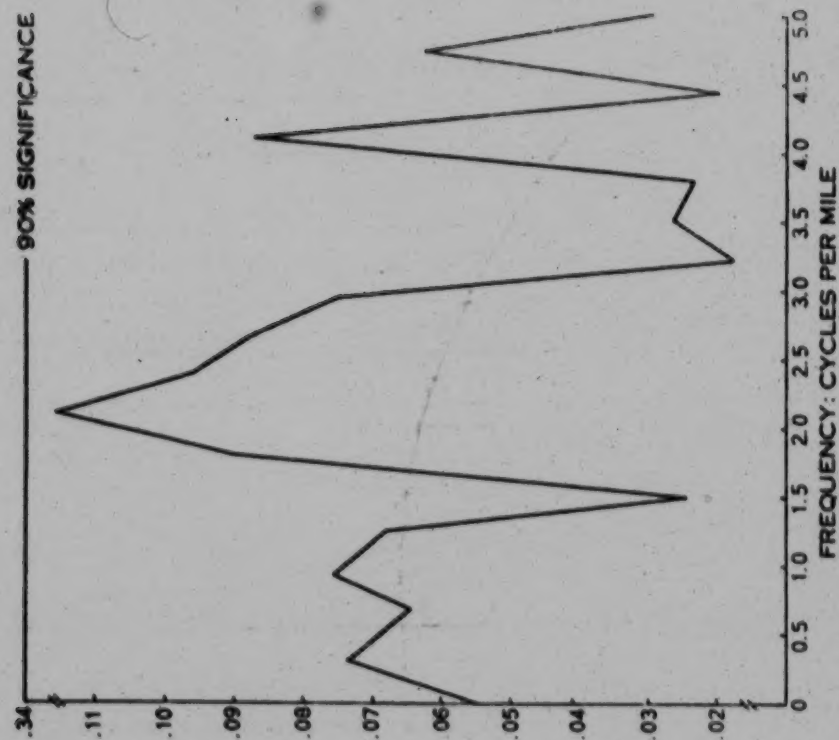


COHERENCE SQUARE ESTIMATES, USING
| ϕ |, FOR BEAVER RIVER STRAIGHT REACH
FIGURE 4.32



COHERENCE SQUARE ESTIMATES, USING
| ϕ |, FOR BEAVER RIVER UPSTREAM
MEANDERS

FIGURE 4.33



COHERENCE SQUARE ESTIMATES, USING
| ϕ |, FOR BEAVER RIVER, ENTIRE REACH

FIGURE 4.34

CHAPTER V - CONCLUSIONS

A. THEORETICAL CONTRIBUTIONS

Classical methods of describing and explaining fluvial meandering were handicapped by subjectivity, partially due their lesser degree of technological and technical sophistication. Until recently, large amounts of data created more problems than they solved. The introduction of new techniques useful for both the collection and subsequent analysis of great quantities of data should prove beneficial to the science of fluvial geomorphology.

The introduction of spectral and cross-spectral analytic techniques into fluvial geomorphology began in 1965 with Speight's work on the Angabunga River, but use of the technique as a descriptive statistic for fluvial systems seems to have advanced little in the past eight years. The technique allows examination of large amounts of data, but long series have not been available until recently. Although it is possible to record, continuously, depths along the path of flow of rivers, this possibility has not been widely exploited in fluvial geomorphology, prior to this study.

A problem arises, however, in comparing spectral studies of meandering to the more traditional studies in the field. Meander wavelength was classically defined as down-valley distance. Spectral and cross-spectral analytic techniques require, for analysis, only single-valued functions. Rivers in nature can seldom be assigned a

baseline or origin allowing them this property in Cartesian space, and so, for spectral analytic studies, series are constructed down-path rather than down-valley. Resulting "wavelengths" represent down-path characteristics that cannot be compared with those found by traditional methods. More precisely, "path length" wavelength, as here defined, varies with wave amplitude, as defined by down-valley Cartesian co-ordinates. The transformation between these two wavelengths is obviously the sinuosity ratio, but this can only be non-arbitrary (and objective) when meander wavelength is known. Thus, when multiple wavelengths co-exist, the sinuosity ratio is arbitrary (subjectively derived), and the transformation cannot exist (without serious bias causing its invalidation).

Wavelengths derived in variance spectrum analysis, however, relate more directly the flow and its effects, as effects are measured along the path the river chooses in sculpturing its fluvial landscape. "Typical" wavelengths, then, are no longer described, but entire stretches of flow path are analyzed as single continuous series. The occurrence of all possible wavelengths within the data is allowed, and even assumed, in spectral analysis, providing for the description of all possible scales of activity, above a size equal to twice the sampling interval, within the series. Relative contributions of different scales of activity are measured and defined.

Intentions underlying the research conducted were focused on the collection and analysis of long, continuous series, sampled at discrete intervals, of plan and depth information for a variety of rivers. The actual study encompassed sand and gravel bed materials; large and small discharges; incised and freely meandering reaches;

braided, meandering, and straight channels; tranquil and turbulent flows; and glaciated and non-glaciated terrains. The research itself encompassed the employment, and to some extent, development, of both data collection and data analysis techniques. Some conclusions may be drawn:

1. It is possible, and indeed desirable, to collect and analyze long records of plan and depth data, in order to attempt the discovery of "meaning" within the data.
2. Boat speed varies along the river during the process of depth recording, and the resulting errors must be corrected. Systematic removal of points, in proportion to speed of recording, provides adequate and satisfactory correction for this problem. Detail can be retained if sampling intervals are sufficiently small.
3. Digitization by machine (Digicon) of depth chart records, and subsequent storage of data on magnetic tape, is an extremely fast, accurate, and economic way of preparing data for analysis. In this study, about eighty-seven feet of depth chart, broken into 203 blocks for subsequent length correction, were digitized with $x = .02$ inches, so that over 52,000 (x,y) co-ordinates were stored, for a total cost, including Cal-comp plotting of the stored data, of about eighty dollars (\$80.).
4. Self-stationarity tests can be easily applied to data and provide more ready communication of information about the series for persons not involved in the research. Comparability of research is thus facilitated.

No previous researchers using spectral techniques for meander or depth forms seem to have tested for stationarity.

Observations made with respect to the data collected using simple first and second moment statistics, and using variance spectrum analysis. Some theories (explained earlier in the review of literature) were tested. Reinforcement of some previous hypotheses via sophisticated statistical analysis, may be of some value to the discipline. Some conclusions may be drawn:

5. Simple first and second moments may be useful in the detection of broad patterns of activity within the data. Underlying processes, such as geologic control, may be suggested by these statistics.
6. Fluvial planform series tend to establish an equilibrium in angular deviation from flow path. Mean angular deviation tends strongly towards zero (0), or complete balance, on rivers with regular meandering, irregularly meandering, straight, incised, and even braided channels.
7. Radius of curvature, based on circular paths, is a versatile statistic for the description of meander planform. It relates planform to flow acceleration in bends, and provides a non-dimensional plan parameter when divided by river width. The parameter R/w shows a strong relationship with the value $1/\pi$ such that $R/\pi w$ appears to vary within narrow limits. Derivation of w makes difficult, in this study, precise definition of the relation, but the range from

1.1 to 2.3 seems to describe the fluctuations of $R/\pi w$ in this study.

8. Both plan and depth spectra indicate a tendency for down-path wavelength to increase in the downstream direction. Coherency estimates support this observation. The trend is not sufficient, however, to make the series non-self-stationary.
9. Both plan and depth spectra indicate a tendency for most pronounced down-path wavelengths to lie within the range of about 6-20 river widths.
10. Coherence wavelengths for the Beaver tend to occur as multiples of the product πw (π times the river width). This is consistent with the findings of Church (1970) and the predictions of Yalin (1971).
11. The Beaver River demonstrated a tendency for down-path wavelengths to approximate a value $\pi \bar{A}$, where $\bar{A}$ is a mean cross-sectional area parameter based on the assumption of a rectangular channel. $\bar{A}$ is calculated as the product of the mean depth and mean width for each reach considered. This finding may be fortuitous, since the relationship was not as strong for the other rivers studied.
12. Spectra of plan and depth and coherence tend to resemble that for a second-order autoregressive process, concentrating power in intermediate frequencies and decreasing in power towards high frequencies. These observations are consistent with the findings of Ashida and Tanaka (1967), Hino (1968), and Nordin and Algert (1968).

13. Down-path wavelengths for plan data tend to occur at values twice that for depth spectra, indicating the persistence of pool-and-riffle sequences in all manner of channels and bed material. The sequence is found for braided, straight, meandering (irregularly and regularly) and incised channels. Coherency spectra support this observation.
14. Coherence is generally low between depth and relative plan data, although some significant coherences occur at greater than 90% probability. In general, coherence is not greater when absolute curvature data are used rather than relative curvature data.
15. Similarities in bed and plan configuration occur for rivers of various bed and bank materials, indicating a more direct relationship of form to flow than to bed and bank media.
16. Spectral and cross-spectral analytic techniques increase objectivity while maximizing quantities of data analyzed, and results can be related both to other variables and to the map. Wavelength is defined as down-path distance, measured along the path of flow. Coherence statistics can provide insight into the relations between form and process.

B. RECOMMENDATIONS FOR FUTURE RESEARCH

The research conducted in the completion of this thesis was in many ways new. New methods of data collection and preparation were attempted, and relatively new methods were employed in the analysis of the data. Many problems were encountered in the endeavour, and though

some were solved in the process of this work, others remain unsolved. Ideas encountered are offered below as recommendations towards research into the broader problem--the mystery of fluvial meandering.

1. Improvement of Type of Data Collected

Accurate measurement of river widths recorded in the field at the time of the depth survey would be useful in the examination of precise relations between hydraulic variables. Widths were obtained from maps and air photographs for this study, and the limits on their accuracy restricted the use of some ideas that seemed promising. Widths could be measured for both bankfull and flood stages.

Velocity measurements could be used in the analysis of relationships between other variables of interest, and they could also be useful in data correction, accounting for boat speed relative to banks.

Accurate measurement of velocity at a variety of depths and cross-river positions would be valuable to an understanding of many physical processes acting within the river to form the depths and meanders observed.

Sediment samples would be of further use in the definition of processes operating in the river, and they would facilitate comparison of research. A knowledge of sediment grain size would broaden the scope of the work.

Depth and plan data should be recorded so that they correspond as precisely as possible. Where possible, thalweg data should be obtained first.

Slope data would also be useful in the examination of fluvial meandering processes. Information on both water surface slope and bed

slope would be extremely valuable.

Data gathered at bank altering stages of flow would be valuable in solution of some of the problems involved in meandering. Ready access to boats and recording equipment is necessary, however, for this to be accomplished.

2. Resolution of Data

Where digitization of data is little problem, as much data as possible should be gathered. Recording charts should be run quickly to maximize information obtained.

In order to preserve maximum resolution of depths in cross-spectral analysis, planform data must be expanded. Interpolation is suggested when analysis is to focus on small-scale processes within the river, although data removal proved most satisfactory toward the examination of large-scale activity in this study.

Optimally, two-dimensional mapping of bedforms would eliminate many problems of the depth survey while expanding scope for data analysis tremendously. Instrumentation exists for continuous mapping, but is not readily available, owing to its expense. In lieu of this degree of sophistication, a number of channel cross-sections could be obtained to improve utility of data and provide some of the information sought. This step of data collection requires time and labor, and thus incurs additional expense.

3. Accuracy of Absolute Location

Absolute location must be maintained as accurately as possible during the depth survey. Air photos and maps are not always sufficient for this purpose. Regular recording of compass bearing and time elapsed

since beginning of survey, with respect to chart position, could help to overcome this problem.

4. Data Analysis

Programs for spectral and cross-spectral analysis must be able to handle vast amounts of data if interpolation methods for retention of resolution succeed. Dr. M. Church at the University of British Columbia (M. Church, pers. comm.) is presently completing work on a program designed to handle up to 100,000 data points in the eventual solution of the problem of meandering.

5. Expansion of Breadth of Research

Data must eventually be gathered and analyzed for rivers of all sizes and channel forms. Co-operation among workers in the field is essential for this to happen. An exciting possibility, based on the work of Yalin (1971), is the concept of obtaining two-dimensional spectra of bedforms, and relating these, via 2-D cross-spectra, to two dimensional water velocity variations. If the hypothesis that differing scales of turbulence produce observed bedforms is correct, then phase spectra, transfer functions, and coherence spectra should provide elucidation of these processes, and the writer plans to do further research in this area.

Applicability of results obtained will be known more precisely as these suggestions are carried out, and prediction of form and process will become more possible.

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APPENDIX I - STATISTICS FOR DEPTH SERIES CORRECTION

(1) SERIES A - South Nahanni River, recording upstream to Virginia Falls

Storage file #	Block #'s included*	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in Corrected file	Remarks
1	1	84	6	7	78	Files 1
2	2	247	32	13	215	to 28
3	3	129	42	42	87	comprise
4	4	121	41	34	80	the section
5	5	108	12	11	96	of braided
6	6	149	44	29	105	channel
7	7	194	62	32	132	below
8	8	97	20	20	77	First
9	9	153	71	46	82	Canyon
10	10	210	76	36	134	
11	11	149	48	32	101	
12	12	115	5	4	110	

* corresponding to Figure 3.03; map of position fixes, where block #1 extends from fix #1 to fix #2

Storage file #	Block #'s included	# Points raw data as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in Corrected file	Remarks
13	13	295	115	39	180	
14	14	138	54	39	84	
15	15	144	56	39	88	
16	16	257	100	39	157	
17	17	152	59	39	93	braids
18	18	114	8	7	106	continue
19	19	683	280	41	403	to block
20	20	95	4	4	91	28
21	21	183	50	27	133	
22	22	148	3	2	145	
23	23	196	9	5	187	
24	24	196	59	30	137	
25	25	114	5	4	109	
26	26	115	34	29	81	
27	28	139	41	29	98	
28	29	157	11	7	146	

Storage file #	Block #'s included	#Points raw data as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in Corrected file	Remarks
29	30	158	46	29	112	First
30	31	119	47	39	72	Canyon
31	32	148	48	32	100	comprised
32	33	251	63	25	188	by blocks
33	34	113	34	30	79	29 - 37
34	35	230	46	20	184	(standard
35	36	80	0	0	80	block)
36	37	182	48	25	144	
37	38	110	38	34	72	
38	39	57	18	32	39	
39	40	60	4	7	56	braids
40	41	178	57	32	121	continue
41	42	192	58	30	134	to file
42	43	255	59	23	196	# 44
43	44	129	48	37	81	
44	45	105	14	13	91	

Storage file #	Block #'s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points removed in correction	# Points left in Corrected file	Remarks
45	46	279	64	23	23	215	
46	47	102	21	20	20	81	2nd Canyon
47	48	52	14	27	27	38	extends
48	49	109	32	29	29	77	from file
49	50	163	44	27	27	119	# 45 to
50	51	97	11	11	11	86	file # 53
51	52	108	17	16	16	91	
52	53	141	41	29	29	100	
53	54	138	32	23	23	106	
54	55	200	28	14	14	172	
55	56	179	70	39	39	109	3rd Canyon
56	57	85	17	20	20	68	extends
57	58	91	18	5	5	73	from file
58	59	43	18	4	4	25	#54 to
59	60	110	26	23	23	84	file # 60B

Storage file #	Block #s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points removed in correction	# Points left in Corrected file	Remarks
60	61	193	27	14	166		
61	62	237	50	32	167		
62	64	181	67	37	114		3rd
63	65	107	27	25	80		
64	66	190	44	23	146		Canyon
65	66B	135	15	11	120		
66	67	182	37	20	145		
67	68	187	26	14	161		
68	69	137	19	14	118		Plains
69	70	211	19	9	192		blocks
70	71	272	87	32	185		# 67
71	72	549	247	45	302		to
72	73	156	50	32	106		# 79
73	74	85	34	39	51		
74	75	88	10	11	78		

Storage file #	Block #'s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points removed in correction	# Points left in corrected file	Remarks
75	76	63	36	57	27		
76	77	110	38	34	72		
77	78	135	37	27	98		
78	79	138	75	54	63		
79	80	161	60	37	101		
80	81	254	122	48	132		
81	82	268	110	41	158		

4th

Canyon

Series total: 81 files, 82 blocks, 9510 points remain after correction for boat speed.

APPENDIX I - STATISTICS FOR DEPTH SERIES CORRECTION

(2) SERIES B - South Nahanni River, recording downstream from Virginia Falls to Liard River Confluence

Storage file #	Block #'s included*	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points removed in correction	# Points left in corrected file	Remarks
82	70	162	31	19	131	Plains	
83	69	102	26	25	76	section	
84	68	128	21	16	107	to block # 67	
85	67	98	0	0	98		
86	66B	80	2	2	78		
87	66	120	22	18	98	3rd Canyon	
88	65	57	3	6	54	blocks	
89	64	101	24	24	77	#66B	
90	63	250	85	34	165	to	
91	62	219	112	51	107	# 54	
92	61	164	71	43	93		
93	60	102	46	45	56		
94	59	50	24	48	26		

* corresponding to Figure 3.03; map of position of fixes, where block #70 extends from fix #70 to fix #71.

Storage File #	Block #'s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
95	58 + 57	152	35	23	117	
96	56	104	33	31	71	
97	55	132	17	13	115	
98	54	97	26	27	71	
99	53	84	17	20	67	
100	52	67	7	10	60	
101	51 + 50	170	31	18	139	2nd Canyon blocks
102	49	72	18	25	54	#55
103	48	40	11	28	29	to
104	47	58	9	16	49	# 46
105	46	182	40	22	142	
106	45	76	10	13	66	
107	44	94	45	47	49	braids
108	43	197	61	31	136	blocks
109	42	121	33	27	88	# 45 to # 39

Storage file #	Block #'s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
110	41	110	31	28	79	
111	40	37	4	10	33	
112	39	74	48	64	26	
113	38 + 37	220	73	33	147	
114	36 + 35	185	13	7	172	
115	34	63	25	40	38	First
116	33	130	8	6	122	Canyon
117	32	86	17	20	69	blocks
118	31	71	22	31	49	# 38
119	30	105	29	28	76	to
120	29	108	11	10	97	# 29
121	28 + 27	237	111	47	126	
122	25 + 24	209	40	19	169	braids
123	23	141	15	11	126	to
124	22	130	32	25	98	confluence

Storage file #	Block #'s included	# Points raw data, digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
125	21	113	25	22	88	
126	20	71	9	12	62	
127	19	390	121	31	269	
128	18	102	31	30	71	braided
129	17 + 13	635	230	36	405	section
130	12	107	33	31	74	continues
131	11	84	14	17	70	to
132	10	108	20	18	88	block
133	9	70	28	35	52	# 1
134	8	64	12	19	52	
135	7	138	49	35	89	
136	6	98	28	29	70	
137	5	65	1	2	64	
138	4	63	12	18	51	
139	3	68	9	13	59	

Storage file #	Block #'s included	# Points raw data, digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
140	2	160	18	11	142	
141	1	77	22	30	52	

Series Total: 60 files, 70 blocks, 5604 points remain after correction for boat speed.

APPENDIX I - STATISTICS FOR DEPTH SERIES CORRECTION

(3) SERIES C - Kootenay River, recording upstream from Canal Flats to Cable Car Crossing

Storage file #	Block #'s included *	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
142	1	337	159	47	178	
143	2	312	122	39	190	
144	3	400	120	30	280	braids
145	4	390	82	21	308	
146	5	171	43	25	128	
147	6	499	234	39	365	
148	7	327	46	14	281	irregular
149	8	815	441	54	374	
150	9	154	68	44	86	meander
151	10	465	238	51	227	
152	11	349	136	39	213	
153	12	225	36	16	189	
154	13	658	290	44	368	

* corresponding to Figure 3.04, map of position fixes, where block #1 extends from fix #1 to fix #2.

Storage file #	Block #'s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points removed in correction	# Points left in corrected file	Remarks
155	14 + 15	830	366		44	464	
156	16	377	162		43	215	
157	17	364	161		44	203	
158	18	683	253		37	430	<u>straights</u>
159	19	963	337		35	626	<u>bend</u>
160	20	586	317		54	269	<u>rock canyon</u>
161	21	1165	595		51	570	
162	22	336	158		47	178	
163	23	402	263		65	139	
164	24	389	265		68	124	Irregular
165	25	795	406		51	389	meanders
166	26	367	121		33	246	continue
167	27	410	173		42	237	to
168	28	616	271		44	345	block
169	29	739	326		44	413	# 37 (end)

Storage file #	Block #'s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
170	30	613	258	42	355	
171	31	463	204	44	259	
172	32	451	0	0	451	(standard block)
173	33	483	223	46	260	
174	34	795	310	39	485	
175	35	250	193	39	57	
176	36	382	80	21	302	
177	37	401	177	44	224	

Series total: files, 37 blocks, 10428 points remain after correction for boat speed.

APPENDIX I - STATISTICS FOR DEPTH SERIES CORRECTION

(4) SERIES D - Beaver River, recording upstream to Beaver Crossing Bridge

Storage file #	Block # ^s included*	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
178	1	606	0	0	606	
179	2	428	107	25	321	meandering
180	3	402	0	0	402	section
181	4	820	361	44	459	to
182	5	184	57	31	127	block
183	6	56	0	0	56	Ø13
184	7	755	144	19	611	
185	8	1051	463	44	588	
186	9	378	177	31	258	
187	10	1299	325	25	974	
188	11	989	248	25	741	
189	12	178	21	12	157	

* corresponds to Figure 3.02, map of position fixes, where block Ø1 extends from fix Ø1 to fix Ø2

Storage file #	Block #'s included	# Points raw data, as digitized @ 1/50"	# Points removed in correction	% Points removed in correction	# Points left in corrected file	Remarks
190	13	1117	212	19	905	
191	14	225	43	19	182	curves
192	15	265	0	0	265	
193	16	2373	736	31	1637	straight
194	18	409	25	6	384	reach
195	19	188	0	0	188	blocks
196	20	669	40	6	629	# 16 - 20
197	21	556	206	37	350	meanders
198	22	1651	99	6	1552	block
199	23	538	102	19	436	# 21
200	24	449	0	0	449	to
201	25	369	70	19	299	end
202	26	800	96	12	704	(standard)
203	27	670	376	56	294	

Series total: 26 files, 27 blocks, 13574 points remain after correction for boat speed

APPENDIX 1

STATISTICS FOR DEPTH SERIES CORRECTION

(5) COMPUTATIONS IN CORRECTION

- Standardization of Series

Step 1. Calculate mean block speed, $\bar{v}$, for each block, as:

$$\begin{aligned}\bar{v} &= \frac{\text{distance travelled}}{\text{time elapsed}} \\ &= \frac{\text{map distance}}{\frac{\text{chart length}}{\text{chart speed}}} = \frac{L}{L/L} = \frac{L}{T}\end{aligned}$$

Step 2. Form ratios of each mean block speed, $\bar{v}_i$, to fastest block speed found, $\bar{v}_{\max}$, as:

$$r = \frac{\bar{v}_i}{\bar{v}_{\max}}$$

Step 3. Remove, via systematic sampling, $100(1-r)$ per cent of the points in each block.

Step 4. Join all points remaining, in sequence, to form the "standardized" depth series.

APPENDIX 2

COMPUTATIONAL PROCEDURE USED IN SPECTRAL

AND CROSS-SPECTRAL ANALYSES OF DATA

(source: BMD:02T program)

COMPUTATIONAL PROCEDURE

1. Symbols Used

- X'_i i^{th} value of discrete, equi-spaced time series $X(t)$
 X_i i^{th} value of time series $X(t)$ after subtracting mean
 Y_i i^{th} value of time series $Y(t)$ after subtracting mean
 Z_i i^{th} value of time series after prewhitening
 p Lag
 C Constant value ($|C| < 1.0$) provided by user
 n Number of discrete data points
 m Maximum lag
 Δt Constant time interval
 $R_x^{(p)}$ Autocovariance of series X at lag p , ($R_x^{(p)} = R_x^{(p\Delta t)}$)
 $A_x^{(p)}$ Autocovariance of series X after detrending at lag p
 $P_x^{(h)}$ Power spectral estimate of series X at frequency
 $\frac{h\pi}{m\Delta t}$ ($P_x^{(h)} = P_x\left(\frac{h\pi}{m\Delta t}\right)$)
 $SP_x^{(h)}$ Smoothed power spectral estimate of series X at
frequency $\frac{h\pi}{m\Delta t}$

$RSP_x^{(h)}$	Power spectral estimate of series X at frequency $\frac{h\pi}{m\Delta t}$ after recoloring
$R_{xy}^{(p)}$	Cross-covariance between series X and Y at lag p
$A_{xy}^{(p)}$	Cross-covariance between series X and Y after detrending at lag p
$P_{xy}^{(h)}$	Cross-power spectral estimate at frequency $\frac{h\pi}{m\Delta t}$
$C_{xy}^{(h)}$	Cospectrum, the real part of $P_{xy}^{(h)}$ at frequency $\frac{h\pi}{m\Delta t}$
$Q_{xy}^{(h)}$	Quadrature spectrum, the imaginary part of $P_{xy}^{(h)}$ at frequency $\frac{h\pi}{m\Delta t}$
$SC_{xy}^{(h)}$	Smoothed cospectrum at frequency $\frac{h\pi}{m\Delta t}$
$SQ_{xy}^{(h)}$	Smoothed quadrature spectrum at frequency $\frac{h\pi}{m\Delta t}$
$AM_{xy}^{(h)}$	Amplitude of cross-spectrum at frequency $\frac{h\pi}{m\Delta t}$
$PHAS_{xy}^{(h)}$	Phase of cross-spectrum at frequency $\frac{h\pi}{m\Delta t}$
$T_{xy}^{(h)}$	Transfer function at frequency $\frac{h\pi}{m\Delta t}$
$TAM_{xy}^{(h)}$	Amplitude of transfer at frequency $\frac{h\pi}{m\Delta t}$
$COSQ_{xy}^{(h)}$	Coherence square at frequency $\frac{h\pi}{m\Delta t}$

2. Procedure

Step 1. The mean of the data is first calculated and subtracted from each value of X' .

$$m_x = \frac{1}{n} \sum_{i=1}^n X'_i$$

$$X_i = X'_i - m_x \quad i = 1, 2, \dots, n$$

This is not done if the series is to be detrended.

Step 2. (optional) One method of prewhitening the input data is by the following moving linear combination:

$$Z_i = X_{i+1} - CX_i, \quad i = 1, 2, \dots, n-1$$

C is a value supplied by the user ($|C| < 1.0$)

Step 3. The autocovariance is then computed.

$$R_x^{(p)} = \frac{1}{n-p} \sum_{q=1}^{n-p} X_q X_{q+p} \quad p = 0, 1, 2, \dots, m$$

$$R_x^{(p)} = R_x(p\Delta t)$$

Step 4. (optional) If the series is to be detrended, a least squares fitting method is used

$$A_x^{(p)} = R_x^{(p)} - \beta - \alpha i \quad \text{where } i = 0, 1, \dots, n-1, \text{ where}$$

$$\alpha = \sum_{i=0}^n x_i (2i-n+1) / (((n-1)n(n+1))/6) \text{ and } \beta = \bar{x} - \alpha(n-1)/2$$

where $\bar{x}$ is the mean.

Step 5. The raw estimate of the power spectrum is obtained by

$$P_x^{(h)} = \frac{2\Delta t}{\pi} \sum_{p=0}^m \epsilon_p R_x^{(p)} \cos \frac{hp\pi}{m} \quad h = 0, 1, \dots, m$$

$$\text{where } \epsilon_p = \begin{cases} 1 & 0 < p < m \\ 1/2 & p = 0, m \end{cases}$$

$$\text{Note: } P_x^{(h)} = P_x(\omega_h) = P_x\left(\frac{h\pi}{m\Delta t}\right)$$

Step 6. The raw estimates of the power spectrum are then smoothed by "hamming".

$$SP_x^{(0)} = .54P_x^{(0)} + .46P_x^{(1)}$$

$$SP_x^{(h)} = .23P_x^{(h-1)} + .54P_x^{(h)} + .23P_x^{(h+1)}, \quad 0 < h < m$$

$$SP_x^{(m)} = .54P_x^{(m)} + .46P_x^{(m-1)}$$

Step 7. A check sum is computed to check the computations of the estimates

$$CHKSUM = \frac{\pi}{m\Delta t} \left[\frac{1}{2} (SP_x^{(0)} + SP_x^{(m)}) + \sum_{h=1}^{m-1} SP_x^{(h)} \right]$$

This should equal $R_x^{(0)}$, the autocovariance at zero lag.

Step 8. To compensate for the prewhitening in Step 2, the smoothed spectrum is recolored by

$$RSP_x^{(h)} = SP_x^{(h)} / (1 + C^2 - 2C \cos \omega_h \Delta t)$$

$$h = 0, 1, 2, \dots, m$$

Similarly, Steps 1 to 8 can be applied to series Y.

Step 9. The cross-covariance is computed by the formula

$$R_{xy}^{(p)} = \frac{1}{n-p} \sum_{q=1}^{n-p} X_q Y_{q+p}, \quad p = 0, 1, 2, \dots, m$$

$$R_{xy}^{(-p)} = \frac{1}{n-p} \sum_{q=1}^{n-p} X_{q+p} Y_q, \quad p = 0, 1, 2, \dots, m$$

Step 10. (optional) In detrending the series

$$A_{xy}^{(I')} = R_{xy}^{(p)} - \beta - \alpha i \text{ where } i = 0, 1, \dots, n-1$$

α and β are defined in Step 4.

Step 11. The cross-spectrum, $P_{xy}^{(h)}$, is given by

$$P_{xy}^{(h)} = C_{xy}^{(h)} + iQ_{xy}^{(h)}$$

The cospectrum is obtained from

$$C_{xy}^{(h)} = \frac{\Delta t}{\pi} \sum_{p=0}^m \epsilon_p \left(R_{xy}^{(p)} + R_{xy}^{(-p)} \right) \cos \frac{hp\pi}{m}$$

The quadrature spectrum is obtained from

$$Q_{xy}^{(h)} = \frac{\Delta t}{\pi} \sum_{p=0}^m \epsilon_p \left(R_{xy}^{(p)} - R_{xy}^{(-p)} \right) \sin \frac{hp\pi}{m}$$

$$h = 0, 1, 2, \dots, m$$

$$\epsilon_p = \begin{cases} 1/2 & p = 0, m \\ 1 & 0 < p < m \end{cases}$$

Step 12. Both the cospectrum and quadrature spectrum are

smoothed by "hamming" as shown in Step 6.

Step 13.

$$AM_{xy}^{(h)} = \sqrt{[SC_{xy}^{(h)}]^2 + [SQ_{xy}^{(h)}]^2}$$

$$PHAS_{xy}^{(h)} = \text{Arg}(SC_{xy}^{(h)} + iSQ_{xy}^{(h)})$$

the amplitude
phase of the
cross-spectrum
are computed

The phase angle computed is the phase shift of series Y with respect to series X.

Step 14. The transfer function from series X to Y is given by

$$T_{xy}^{(h)} = \frac{P_{xy}^{(h)}}{P_x^{(h)}} = TAM_{xy}^{(h)} e^{iPHAS_{xy}^{(h)}}$$

The amplitude is given by

$$TAM_{xy}^{(h)} = AM_{xy}^{(h)} / SP_x^{(h)}$$

The phase shift from X to Y is the same as that of the cross-spectrum. Both the transfer function amplitudes from X to Y and Y to X are printed.

Step 15. Finally the coherence square, between X and Y, is given as

$$COSQ_{xy}^{(h)} = \left(AM_{xy}^{(h)} \right)^2 / SP_x^{(h)} \cdot SP_y^{(h)}$$

APPENDIX 3

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